EXERCISE NO: 10.1

Question 1: How many tangents can a circle have?

Solution. A circle can have an infinite number of tangents.

Question 2: Fill in the blanks :
(i) A tangent to a circle intersects it in _________ point(s).
(ii) A line intersecting a circle in two points is called a _______.
(iii) A circle can have _______ parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called ______.

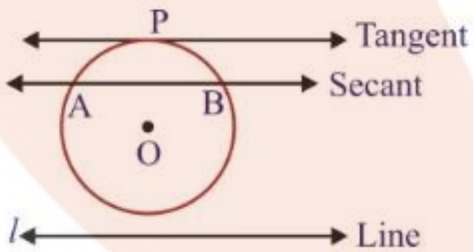
Solution 2:
(i) A tangent to a circle intersects it in exactly one point(s).
(ii) A line intersecting a circle in two points is called a secant.
(iii) A circle can have two parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called Point of Contact.

Question 3: A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length of PQ is :
(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$

Solution 3:
(D) is the answer.
Because, PQ = $\sqrt{(OQ^2 - OP^2)} = \sqrt{(12^2 - 5^2)} = \sqrt{144-25} = \sqrt{119}$

Question 4: Draw a circle and two lines parallel to a given line such that one is tangent and the other, a secant to the circle.

Solution 4:
From the Given Figure below,

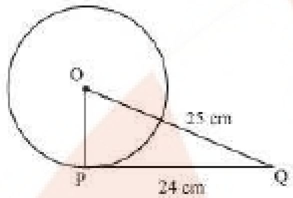


Let ‘l’ be the given line and a circle with centre O is drawn.
- Line PT is drawn || to line ‘l’
- PT is the tangent to the circle.
- AB is drawn || to line ‘l’ and is the secant.

EXERCISE NO: 10.2

Question 1:
From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
(A) 7 cm (B) 12 cm
(C) 15 cm (D) 24.5 cm

Solution 1:

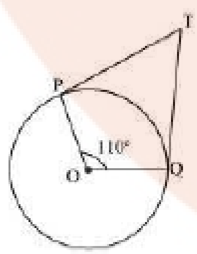


Let 'O' be the centre of the circle
Given:

- Distance of Q from the centre, OQ = 25cm
- Length of the tangent to a circle, PQ = 24 cm
- Radius, OP = ?

We know that, Radius is perpendicular to the tangent at the point of contact
Hence, $OP \perp PQ$
Therefore, OPQ forms a Right Angled Triangle
Applying Pythagoras theorem for ΔOPQ,
$OP^2 + PQ^2 = OQ^2$
By substituting the values in the above Equation,
$OP^2 + 24^2 = 25^2$
$OP^2 = 625 - 576$ (By Transposing)
$OP^2 = 49$
$OP = 7$ (By Taking Square Root)
Therefore, the radius of the circle is 7 cm.
Hence, alternative (A) is correct.

Question 2:
In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to
(A) 60° (B) 70° (C) 80° (D) 90°



Solution 2:
Given:

- Tangents: TP and TQ
  We know that, Radius is perpendicular to the tangent at the point of contact
  Thus, $OP \perp TP$ and $OQ \perp TQ$

- Since the Tangents are Perpendicular to Radius
    - $\angle OPT = 90^o$
    - $\angle OQT = 90^o$

Now, POQT forms a Quadrilateral

We know that, Sum of all interior angles of a Quadrilateral = 360°

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360°$

$\Rightarrow 90° + 110^o + 90° + PTQ = 360°$ (By Substituting)

$\Rightarrow \angle PTQ = 70°$

Hence, alternative (B) is correct.

Question 3:

If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then $\angle POA$ is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

Solution 3:

Given:

- Tangents are PA and PB
  We know that, Radius is perpendicular to the tangent at the point of contact
  Thus, $OA \perp PA$ and $OB \perp PB$
- Since the Tangents are Perpendicular to Radius
    - $\angle OBP = 90^o$
    - $\angle OAP = 90^o$

Now, AOBP forms a Quadrilateral

We know that, Sum of all interior angles of a Quadrilateral = 360°

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$

$90° + 80° + 90^o + BOA = 360°$ (By Substituting)

$\angle BOA = \angle AOB = 100°$

In ΔOPB and ΔOPA,

AP = BP (Tangents from a point)

OA = OB (Radii of the circle)

OP = OP (Common side)

Therefore, $\Delta OPB \cong \Delta OPA$ (SSS congruence criterion)

$A \leftrightarrow B,\ P \leftrightarrow P,\ O \leftrightarrow O$

And thus, $\angle POB = \angle POA$

$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

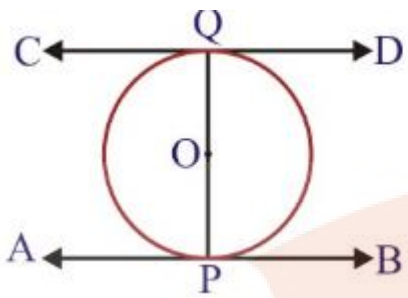
Hence, alternative (A) is correct.

Question 4:

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution 4:

From the figure,



Given

- Let PQ be a diameter of the circle.
- Two tangents AB and CD are drawn at points P and Q respectively.

To Prove:

Tangents drawn at the ends of a diameter of a circle are parallel.

Proof:

We know that, Radius is perpendicular to the tangent at the point of contact

Thus, $OP \perp AB$ and $OQ \perp CD$

Since the Tangents are Perpendicular to Radius

- $\angle OQC = 90^o$
- $\angle OQD = 90^o$
- $\angle OPA = 90^o$
- $\angle OPB = 90^o$

From Observation,

- $\angle OPC = \angle OQB$ (Alternate interior angles)
- $\angle OPD = \angle OQA$ (Alternate interior angles)

If the Alternate interior angles are equal then lines AB and CD should be parallel.

We know that AB & CD are the tangents to the circle.

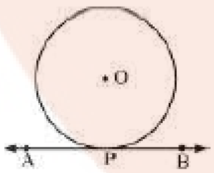
Hence, it is proved that Tangents drawn at the ends of a diameter of a circle are parallel.

Question 5:

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution 5:

From the figure,



Given:

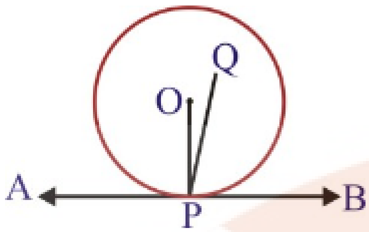
- Let 'O' be the centre of the circle
- Let AB be a tangent which touches the circle at P.

To Prove:

- Line perpendicular to AB at P passes through centre O.

Proof:

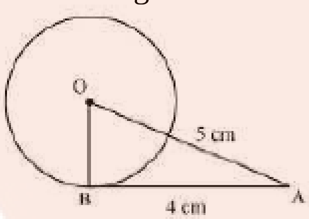
Consider the figure below,



Let us assume that the perpendicular to AB at P does not pass through centre O.
Let It pass through another point Q. Join OP and QP.
We know that, Radius is perpendicular to the tangent at the point of contact
Hence, AB ⊥ PQ
$\therefore \angle QPB = 90°$ … (1)
We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.
$\therefore \angle OPB = 90°$ … (2)
Comparing equations (1) and (2), we obtain
$\therefore \angle QPB = \angle OPB$ … (3)
From the figure, it can be observed that,
$\therefore \angle QPB < \angle OPB$ … (4)
Therefore, in reality $\angle QPB \neq \angle OPB$
$\angle QPB = \angle OPB$ only if QP = OP which is possible in a scenario when the line QP coincides with OP.
Hence it is proved that the perpendicular to AB through P passes through centre O.

Question 6:
The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Solution 6:
From the Figure:



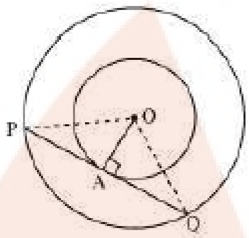
Given:
- Let point ‘O’ be the centre of a circle
- AB is a tangent drawn on this circle from point A, AB = 4 cm
- Distance of A from the centre, OA = 5cm
- Radius, OB = ?

In ΔABO,
We know that, OB ⊥ AB (Radius ⊥ tangent at the point of contact)
OAB forms a Right Angled Triangle.
Hence using, Pythagoras theorem in ΔABO,
$AB^2 + OB^2 = OA^2$
$4^2 + OB^2 = 5^2$ (By Substituting)
$16 + OB^2 = 25$
$OB^2 = 9$

Radius, OB = 3 (By Taking Square Roots)
Hence, the radius of the circle is 3 cm.

Question 7:
Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution 7:
From the Figure,

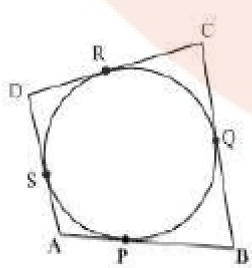


Given,

- Let 'O' be the centre of the two concentric circles
- Let PQ be the chord of the larger circle which touches the smaller circle at point A.
- PQ = ?

By Observation,
Line PQ is tangent to the smaller circle.
Hence, OA ⊥ PQ (Radius ⊥ tangent at the point of contact)
ΔOAP forms a Right Angled Triangle
By applying Pythagoras theorem in ΔOAP,
$OA^2 + AP^2 = OP^2$
$3^2 + AP^2 = 5^2$ (By Substituting)
$9 + AP^2 = 25$
$AP^2 = 16$
AP = 4 (By Taking Square Roots)
In ΔOPQ,
Since OA ⊥ PQ,
AP = AQ (Perpendicular from the center of the circle bisects the chord)
∴ PQ = 2 times AP = 2 × 4 = 8 (Substituting AP = 4cm)
Therefore, the length of the chord of the larger circle is 8 cm.

Question 8:
A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that AB + CD = AD + BC



Solution 8:

From the Figure,
Given,

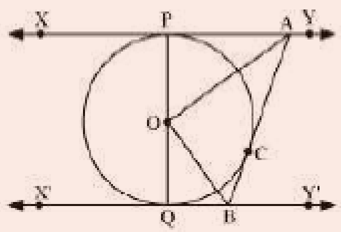
- DC , DA, BC, AB are sides of the Quadrilaterals which also form the tangents to the circle inscribed within Quadrilateral ABCD

To Prove:
AB + CD = AD + BC
Proof:
We know that length of tangents drawn from an external point of the circle are equal.
DR = DS ………..… (1)
CR = CQ ………..… (2)
BP = BQ ………….. (3)
AP = AS ………..… (4)
Adding (1), (2), (3), (4), we obtain
DR + CR + BP + AP = DS + CQ + BQ + AS
(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ) (By regrouping) ------------- (5)
From the figure,

- DR +CR = DC
- BP + AP = AB
- DS + AS = AD
- CQ +BQ = BC

Hence substituting the above values in Equation (5),
CD + AB = AD + BC
Hence it is proved.

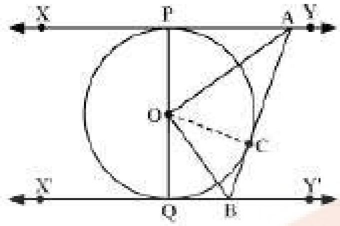
Question 9:
In the given figure, XY and X’Y’ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X’Y’ at B. Prove that ∠AOB = 90°.



Solution 9:
From the Figure,
Given,

- Let ‘O’ be the centre of the circle
- XY and X’Y’ are two parallel tangents to circle
- AB is another tangent such that with point of contact C intersecting XY at A and X’Y’ at B.

To Prove:
∠AOB=90°.
Proof:
Join point O to C.



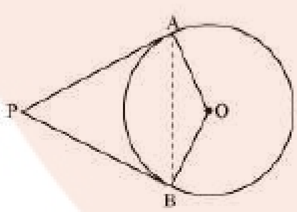
From the Figure above,
Consider ΔOPA and ΔOCA,
Here,

- OP = OC (Radii of the same circle)
- AP = AC (Tangents from external point A)
- AO = AO (Common side)

Therefore, ΔOPA ≅ ΔOCA (SSS congruence criterion)
Hence, P ↔ C, A ↔ A, O ↔ O
We can also say that,
$\angle POA = \angle COA$ … (*i*)
Similarly, ΔOQB ≅ ΔOCB
$\angle QOB = \angle COB$ … (*ii*)
Since POQ is a diameter of the circle, it is a straight line.
Therefore, $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$--------------- (3)
Substituting Equation (i) and (ii) in the Equation (3),
$2\angle COA + 2\angle COB = 180^\circ$
$2(\angle COA + \angle COB) = 180^\circ$
$\angle COA + \angle COB = 90^\circ$ (By Transposing)
$\angle AOB = 90^\circ$

Question 10:
Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

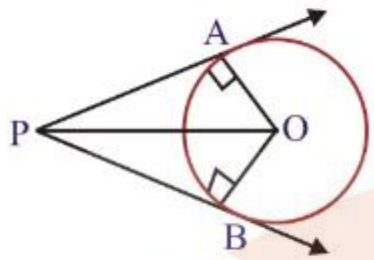
Solution 10:



From the figure,
Given,

- Let us consider a circle centered at point O.
- Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively
- AB is the line segment, joining point of contacts A and B together such that it subtends ∠AOB at center O of the circle.

To Prove:
∠APB is supplementary to ∠AOB.
Proof:
Join OP



Consider the ΔOAP & ΔOBP,

- PA = PB(Tangents drawn from an external point are equal)
- OA = OB (Radii of the same circle)
- OP = OP(Common Side)

Therefore, ΔOAP ≅ ΔOBP (SSS congruence criterion)
Hence,

- $\angle OPA = \angle OPB$
- $\angle AOP = \angle BOP$

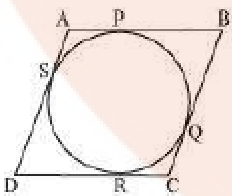
Also,

- $\angle APB = 2\angle OPA$ -------------(1)
- $\angle AOB = 2\angle AOP$--------------(2)

In the Right angled Triangle ΔOAP,
$\angle AOP + \angle OPA = 90^o$
$\angle AOP = 90^o - \angle OPA$ ----------------- (3)
Multiplying the Equation (3) by 2,
$2\angle AOP = 180^o - 2\angle OPA$
By Substituting (1) and (2) in the equation above,
$\angle AOB = 180^o - \angle APB$
$\angle AOP + \angle OPA = 180^o$
Hence it is proved that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Question 11:
Prove that the parallelogram circumscribing a circle is a rhombus.

Solution 11:
From the figure,



Given,

- ABCD is a parallelogram,
- Hence
  - AB = CD …(1)
  - BC = AD …(2)

To Prove:

- Parallelogram circumscribing a circle is a rhombus.

Proof:

From the figure,

- DR = DS (Tangents on the circle from point D)
- CR = CQ (Tangents on the circle from point C)
- BP = BQ (Tangents on the circle from point B)
- AP = AS (Tangents on the circle from point A)

Adding all the above equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ) (By Rearranging) ------ (3)

From the figure,

- DR + CR = CD,
- (BP + AP) = AB
- (DS + AS) = AD
- (CQ + BQ) = BC

Substituting the above values in (3),

CD + AB = AD + BC ------------- (4)

On putting the values of equations (1) and (2) in the equation (4), we obtain

2AB = 2BC

AB = BC ……………………… (5)

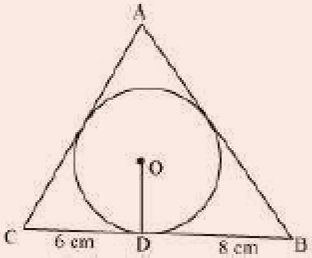
Comparing equations (1), (2), and (5), we get

AB = BC = CD = DA satisfies the property of Rhombus.

Hence, ABCD is a rhombus.

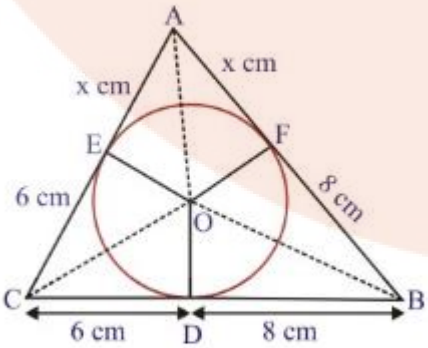
Question 12:

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the Segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.



Solution 12:

From the figure,



Given,

- Let the given circle touch the sides AB and AC of the triangle at point E and F respectively
- Length of the line segment AF be x.
- In ΔABC,
    - CF = CD = 6cm (Tangents on the circle from point C)
    - BE = BD = 8cm (Tangents on the circle from point B)
    - AE = AF = x (Tangents on the circle from point A)
    - AB = ?
    - AC = ?

In ΔABE,
AB = AE + BE = $x + 8$
BC = BD + DC = 8 + 6 = 14
CA = CF + AF = $6 + x$
We know that, $2s$ = AB + BC + CA
$= x + 8 + 14 + 6 + x$
$= 28 + 2x$
$s = 14 + x$
We also known that,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\{14+x\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}$

$= \sqrt{(14+x)(x)(8)(6)}$

$= 4\sqrt{3(14x+x^2)}$

Area of $\Delta OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$

Area of $\Delta OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$

Area of $\Delta OAB = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16x + 2x$

$\Rightarrow 4\sqrt{3(14x+x^2)} = 56 + 4x$

$\Rightarrow \sqrt{3(14x+x^2)} = 14 + x$

$\Rightarrow 3(14x+x^2) = (14+x)^2$

$\Rightarrow 42x + 3x^2 = 196 + x^2 + 28x$

$\Rightarrow 2x^2 + 14x - 196 = 0$

$\Rightarrow x^2 + 7x - 98 = 0$

$\Rightarrow x^2 + 14x - 7x - 98 = 0$

$\Rightarrow x(x+14) - 7(x+14) = 0$

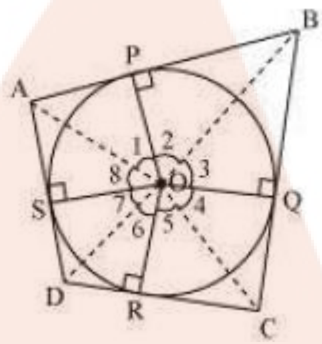
$\Rightarrow (x+14)(x-7) = 0$

Either $x+14 = 0$ or $x - 7 = 0$

Therefore, $x = -14$ and 7
However, $x = -14$ is not possible as the length of the sides will be negative.
Therefore, $x = 7$
Hence, AB = $x + 8 = 7 + 8 = 15$ cm
CA = $6 + x = 6 + 7 = 13$ cm

Question 13:
Prove that opposite sides of a quadrilateral circumscribing a circle subtend Supplementary angles at the centre of the circle.

Solution 13:
From the Figure,



Given,
- Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S.

To Prove:
- Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
- i.e., $\angle AOB + COD = 180^o$ & $\angle BOC + \angle DOA = 180^o$

Proof:
- Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,
AP = AS (Tangents from the same point)
OP = OS (Radii of the same circle)
OA = OA (Common side)
ΔOAP ≅ ΔOAS (SSS congruence criterion)
Therefore, A ↔ A, P ↔ S, O ↔ O
And thus, $\angle POA = \angle AOS$
$\angle 1 = \angle 8$
Similarly,
$\angle 2 = \angle 3$
$\angle 4 = \angle 5$
$\angle 6 = \angle 7$
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$
$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^o$ (By Rearranging)
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^o$
$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^o$
$(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 180^o$

$\angle AOB + \angle COD = 180^\circ$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend Supplementary angles at the centre of the circle.