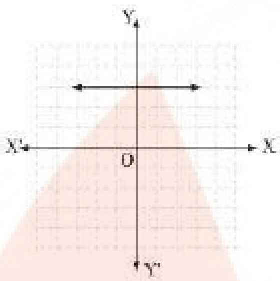
EXERCISE NO: 2.1

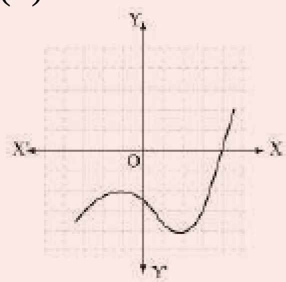
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some Polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

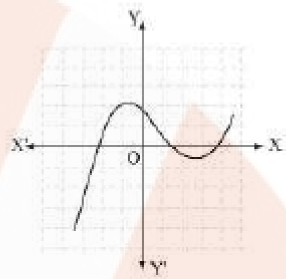
(i)



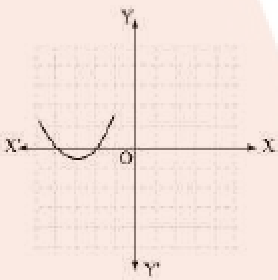
(ii)



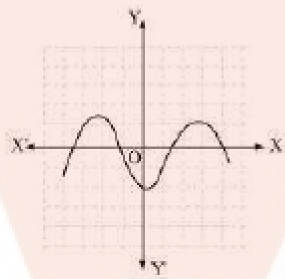
(iii)



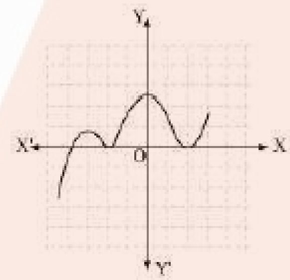
(iv)



(v)



(vi)



Solution 1:

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.
(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.
(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.
(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.
(vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

EXERCISE NO: 2.2

Question 1:
Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) x^2-2x-8 (ii) $4s^2-4s+1$ (iii) $6x^2-3-7x$
(iv) $4u^2+8u$ (v) t^2-15 (vi) $3x^2-x-4$

Solution 1:
(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when $x-4=0$ or $x+2=0$, i.e., when $x=4$ or $x=-2$

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4x(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2-4s+1$ is zero when $2s-1=0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2-4s+1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1\frac{(-4)}{4}=\frac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$

Product of zeroes $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero when $3x+1=0$ or $2x-3=0$, i.e.,

$x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2-3-7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(iv) $4u^2 + 8u = 4u^2 + 8u + 0$

$= 4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

$$\text{Sum of zeroes} = 0 + (-2) = -2 = \frac{(-8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

$$\text{Product of zeroes} = 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

(v)

$t^2 - 15$

$= t^2 = 0t - 15$

$= (t - \sqrt{15})(t + \sqrt{15})$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are and $\sqrt{15}$ and $-\sqrt{15}$.

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$$

$$\text{Product of zeroes} = (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(vi) $3x^2 - x - 4$

The value of $3x2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e., when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}+(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) 0, 5 (iv) 1, 1 (v) $-\frac{1}{4},\frac{1}{4}$

(vi) 4, 1

Solution 2:

(i) $\frac{1}{4},-1$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$$

$$\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$$

If a = 4, then b = −1, c = -4

Therefore, the quadratic polynomial is $4x^2-x-4$.

(ii) $\sqrt{2},\frac{1}{3}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\sqrt{2}=\frac{3\sqrt{2}}{3}=\frac{-b}{a}$$

$$\alpha\beta=\frac{1}{3}=\frac{c}{a}$$

If a = 3, then b = $-3\sqrt{2}$, c = 1

Therefore, the quadratic polynomial is $3x^2-3\sqrt{2}x+1$.

(iii) 0, $\sqrt{5}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=0=\frac{0}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=\sqrt{5}=\frac{\sqrt{5}}{1}=\frac{c}{a}$$

If a = 1, then b = 0, c = $\sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$

If a = 1, then b = -1, c = 1

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4},\frac{1}{4}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{-1}{4}=\frac{-b}{a}$$

$$\alpha\times\beta=\frac{1}{4}=\frac{c}{a}$$

If a = 4, then b = 1, c = 1

Therefore, the quadratic polynomial is $4x^2+x+1$.

(vi) 4, 1

Let the polynomial be ax^2+bx+c.

$$\alpha+\beta=4=\frac{4}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$

If a = 1, then b = −4, c =1

Therefore, the quadratic polynomial is x^2-4x+1.

EXERCISE NO: 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x)=x^3-3x^2+5x-3, g(x)=x^2-2$

(ii) $p(x)=x^4-3x^2+4x+5,\ g(x)=x^2+1-x$

(iii) $p(x)=x^4-5x+6, g(x)=2-x^2$

Solution 1:

(i)

$p(x)=x^3-3x^2+5x-3,$

$g(x)=x^2-2$

$$
\begin{array}{r|l}
 & \qquad\qquad\quad x-3 \\
\cline{2-2}
x^2-2 & x^3-3x^2+5x-3 \\
 & x^3 \qquad\quad -2x \\
 & - \qquad\qquad + \\
\cline{2-2}
 & \quad -3x^2+7x-3 \\
 & \quad -3x^2 \qquad +6 \\
 & \quad\ \ + \qquad\quad - \\
\cline{2-2}
 & \qquad\qquad 7x-9 \\
\cline{2-2}
\end{array}
$$

Quotient = $x-3$

Remainder = $7x-9$

(ii) $p(x)=x^4-3x^2+4x+5=x^4+0.x^3-3x^2+4x+5$

$g(x)=x^2+1-x=x^2-x+1$

$$\begin{array}{r|l} & x^2 + x - 3 \\ \hline x^2 - x + 1 & x^4 \qquad - 3x^2 + 4x - 5 \\ & \underset{-}{x^4} \underset{+}{-} x^3 \underset{-}{+x^2} \\ \hline & \qquad x^3 - 4x^2 + 4x + 5 \\ & \qquad \underset{-}{x^3} \underset{+}{-} x^2 \underset{-}{+} x \\ \hline & \qquad\qquad -3x^2 + 3x + 5 \\ & \qquad\qquad \underset{+}{-}3x^2 \underset{-}{+} 3x \underset{+}{-} 3 \\ \hline & \qquad\qquad\qquad 8 \end{array}$$

Quotient = $x^2 + x - 3$
Remainder = 8

(iii)

$p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$\begin{array}{r|l} & -x^2 - 2 \\ \hline -x^2 + 2 & x^4 + 0.x^2 - 5x + 6 \\ & x^4 - 2x^2 \\ & - \quad + \\ \hline & \quad 2x^2 - 5x + 6 \\ & \quad 2x^2 - 4 \\ & \quad - \quad + \\ \hline & \qquad -5x + 10 \\ \hline \end{array}$$

Quotient = $-x^2 - 2$
Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^2 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Solution 2:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$t^2 - 3 = t^2 + 0.t - 3$

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 + 0.t2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0.t^3 - 6t^2 \\
 & - \quad - \quad + \\
\hline
 & 2t^3 + 4t^2 - 9t - 12 \\
 & 3t^3 + 0.t^2 - 9t \\
 & - \quad - \quad + \\
\hline
 & 4t^2 + 0.t - 12 \\
 & 4t^2 + 0.t - 12 \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$
\begin{array}{r|l}
 & 3x^2 + 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & \overline{\quad -4x^3 - 10x^2 + 2x + 2} \\
 & \quad -4x^3 - 12x^2 - 4x \\
 & \overline{\qquad 2x^2 + 6x + 2} \\
 & \qquad 2x^2 + 6x + 2 \\
 & \overline{\qquad \text{x} \quad \text{x} \quad \text{x}}
\end{array}
$$

Since the remainder is 0,

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^2 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^2 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & \overline{\quad -x^3 \qquad + 3x + 1} \\
 & \quad -x^3 \qquad + 3x - 1 \\
 & \overline{\qquad\qquad 2}
\end{array}
$$

Since the remainder $\neq 0$,

$x^2 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Question 3:

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Solution 3:

$p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\sqrt{\frac{5}{3}}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0.x^3-5x^2 \\
 & -\quad -\quad + \\
\hline
 & 6x^3+3x^2-10x-5 \\
 & 6x^3+0x^2-10x \\
 & -\quad -\quad + \\
\hline
 & 3x^2+0x-5 \\
 & 3x^2+0x-5 \\
 & -\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$$

$$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x+1=0$

$x=-1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x=-1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1.

Question 4:

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$, respectively. Find $g(x)$.

Solution 4:

$p(x)=x^3-3x^2+x+2$ (Dividend)

g(x) = ? (Divisor)

Quotient = $(x-2)$

Remainder = $(-2x+4)$

Dividend = Divisor × Quotient + Remainder

$$x^3-3x^2+x+2=g(x)\times(x-2)+(-2x+4)$$

$$x^3-3x^2+x+2+2x-4=g(x)(x-2)$$

$$x^3-3x^2+3x-2=g(x)(x-2)$$

$g(x)$ is the quotient when we divide (x^3-3x^2+3x-2) by $(x-2)$

$$
\begin{array}{r|l}
x-2 & \overline{x^3-3x^2+3x-2} \quad x^2-x+1 \\
 & x^3-2x^2 \\
 & - \quad + \\
\hline
 & -x^2+3x-2 \\
 & -x^2+2x \\
 & + \quad - \\
\hline
 & x-2 \\
 & x-2 \\
 & - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$$\therefore g(x)=\left(x^2-x+1\right)$$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

Solution 5:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2+2x+2$ by 2.
Here, $p(x)=6x^2+2x+2$
$g(x) = 2$
$q(x) = 3x^2+x+1$ and $r(x) = 0$
Degree of $p(x)$ and $q(x)$ is the same i.e., 2.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
$$6x^2+2x+2=2\left(3x^2+x+1\right)$$
$$=6x^2+2x+2$$
Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$
Let us assume the division of $x^3+ x$ by x^2,
Here, $p(x) = x^3+ x$
$g(x) = x^2$
$q(x) = x$ and $r(x) = x$
Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
$x^3+ x = (x^2) \times x + x$
$x^3+ x = x^3+ x$
Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$
Degree of remainder will be 0 when remainder comes to a constant.
Let us assume the division of $x^3+ 1$ by x^2.
Here, $p(x) = x^3+ 1$
$g(x) = x^2$
$q(x) = x$ and $r(x) = 1$
Clearly, the degree of $r(x)$ is 0.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
$x^3+ 1 = (x^2) \times x + 1$
$x^3+ 1 = x^3+ 1$
Thus, the division algorithm is satisfied.

EXERCISE NO: 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, 2$ $2x^3 + x^2 – 5x + 2$; ½, 1, -2

(ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Solution 1:

(i)p(x) = $2x^3 + x^2 - 5x + 2$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= 0$$

$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$

$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= -16 + 4 + 10 + 2 = 0$$

Therefore, $\frac{1}{2}$, 1, and −2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain *a* = 2, *b* = 1, *c* = −5, *d* = 2

We can take $\alpha = \frac{1}{2}, \beta = 1, y = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1

$p(2) = 2^3 - 4(2^2) + 5(2) - 2$

$= 8 - 16 + 10 - 2 = 0$

$p(1) = 1^3 - 4(1^2) + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 0$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.

Verification of the relationship between zeroes and coefficient of the given polynomial

Sum of zeroes $= 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1)

$= 2 + 1 + 2 = 5 = \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = $2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Solution 2:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta,$ and γ.

It is given that

$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If $a = 1$, then $b = -2$, $c = -7$, $d = 14$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

Solution 3:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are $a - b$, $a + a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$p = 1$, $q = -3$, $r = 1$, $t = 1$

Sum of zeroes = a – b + a + a + b

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are 1 – b, 1 + b.

Multiplication of zeroes = 1(1 – b)(1 + b)

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Question 4:

It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Solution 4:

Given that 2 + $\sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}$$

Clearly, $= x^4 - 6x^3 - 26x^2 + 138x - 35 = \left(x^2 - 4x + 1\right)\left(x^2 - 2x - 35\right)$

It can be observed that $\left(x^2 - 2x - 35\right)$ is also a factor of the given polynomial.

And $= \left(x^2 - 2x - 35\right) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when or $x - 7 = 0$

Or $x + 5 = 0$

Or $x = 7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x - 10$ is divided by another Polynomial $x^2 - 2x + k$, the remainder comes out to be *x* + *a*, find *k* and *a*.

Solution 5:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x - 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide by $x^4 - 6x^3 + 16x^2 - 26x - 10 - a$ by $x^2 - 2x + k$

$$\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 26x \\
 & -4x^3 + \quad 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26-4k)x + 10 - a \\
 & (8-k)x^2 - (16-2k)x + (8k - k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x + (10 - a - 8k + k^2)
\end{array}$$

$$(x^2 - 4x + 1)(x^2 - 2x - 35) = (x-7)(x+5)$$

It can be observed that $(-10+2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10+2k) = 0$ and $(10 - a - 8k + k^2) = 0$

For $(-10+2k) = 0$,

2 *k* =10

And thus, $k = 5$

For $\left(10 - a - 8k + k^2\right) = 0$

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$

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