# E1 ELECTRIC FIELDS AND CHARGE

## OBJECTIVES

### Aims

In studying this chapter you should aim to understand the basic concepts of electric charge and field and their connections. Most of the material provides illustrations which should help you to acquire that understanding. To test whether you have understood the concepts see if you can describe and explain the applications in your own words.

### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms:

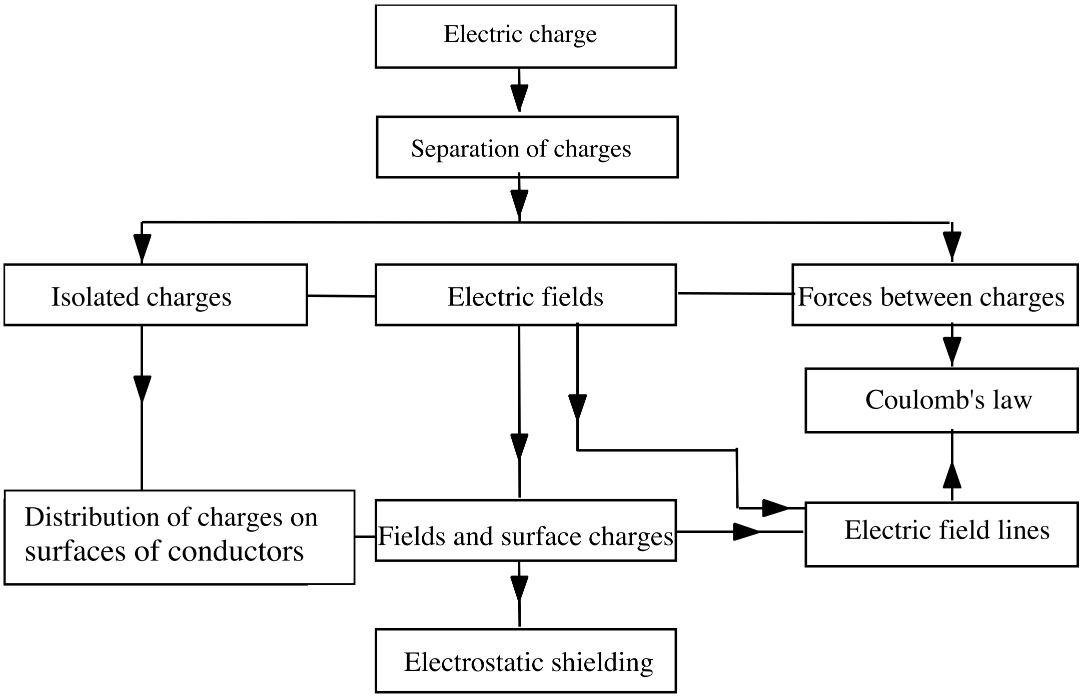
   *electric charge, coulomb, electron, electronic charge, conservation of charge, ion, electrostatic force, permittivity, permittivity of free space, conduction, electric field, field lines, field strength, uniform field, surface charge density, electrical shielding.*

2. Describe a number of processes by which electric charges can be separated.
3. Describe the qualitative features of the interactions of static charges.
4. State and apply the relation between electric force and electric field.
5. Describe and explain examples of applications and hazards of electrostatic phenomena.
6. (a) Sketch electrostatic field-line diagrams for systems with simple conductor shapes including parallel plates, concentric cylinders, isolated sphere, parallel cylinders.

   (b) Interpret given field-line diagrams in terms of magnitude and direction of the field.

7. State and apply the relation between the surface change density and electric field at the surface of a conductor.
8. Describe and discuss principles and examples of electrostatic shielding.

### CONCEPT DIAGRAM



# PRE-LECTURE

## 1-1 INTRODUCTION

The natural world provides precious few clues to the ubiquity of electromagnetism. The properties of rubbed amber and the magnetism of lodestone were little more than curiosities until the 17th century. The elucidation of the nature of lightning and the recognition of light as electromagnetic radiation were even more recent. Yet our world is governed by electromagnetic interactions. All chemical bonds, the strength and weakness of materials, light, the influences that form and control living things are all aspects of electromagnetism. Indeed, only three other types of fundamental force are known: the *weak force*, the *strong force* and *gravity*.

Of these forces the first two, which are purely nuclear forces, are effective only over distances of less than $10^{-12}$ m - that is smaller than an atom. They are associated with radioactivity, but otherwise are of no consequence in chemistry and biology. Gravity is such a feeble force that it matters only in interactions involving bodies of ‘astronomical’ mass and is negligible in interactions between small things. Apart from the earth's gravitational pull, every force that you experience is electromagnetic in nature.

Electromagnetic force is associated with a fundamental property of matter - electric **charge**. No process is known which creates or destroys even the minutest amount of charge. This is formally stated as **the law of charge conservation**: *electric charge can not be created or destroyed.*

The SI unit of electric charge is the coulomb, symbol C. The magnitude of the charge on a single electron is a commonly used quantity in atomic physics. It is

$$e \equiv 1.6 \times 10^{-19}\ \mathrm{C}.$$

Thus $$1\ \mathrm{C} \equiv 6.2 \times 10^{18}\ e.$$

The charge on an electron is $-e$, and on a proton, $+e$.

# LECTURE

## 1-2 ELECTRIC CHARGE

### Why study electrostatics?

Electrostatics is the basis of industrial processes such as electrostatic spray painting, and xerography, and of the unintentional build-up of electrostatic charge which can trigger explosions and fires in, for example, grain silos and operating theatres.

We study electrostatic phenomena in this course not simply to understand the applications of static charge, or the hazards they present, but because the ideas are applicable in other branches of electricity, and the electrical aspects of other sciences. The theory of electrostatics, electric charge, field and potential apply in more complex situations involving currents and magnetic fields.

### Isolated electric charges and atomic structure

#### Demonstration

> Isolated electric charges, such as those obtained by running a comb through your hair, can be detected with a *field mill*, an instrument which measures the charge induced on an internal sensing electrode. A field mill shows that the positive charge on the comb is balanced by a negative charge left behind on the hair.

Electrostatic phenomena are a consequence of the atomic nature of matter. All atoms are built up from three types of fundamental particles, protons, neutrons, and electrons, the properties of which are given in table 1.1. The nucleus of an atom consists of a number of protons and neutrons tightly bound together and surrounded by a cloud of electrons. The number of electrons in the electron cloud is equal to the number of protons in the nucleus. Since the charges of the proton and electron are exactly equal in magnitude and opposite in sign, atoms are electrically neutral. Atoms may gain or lose electrons, thus becoming **ions**.

| **Particle** | **Mass** | **Charge** |
|---|---|---|
| Proton | $m_p$ | $+ e$ |
| Neutron | $1.01\ m_p$ | 0 |
| Electron | $\frac{1}{1840}\ m_p$ | $- e$ |

**Table 1.1 The fundamental particles of atomic physics**

Atoms combine to form molecules, which in turn combine to form bulk material. The normal state of matter is electric neutrality: matter normally contains equal numbers of protons and electrons, and has no net charge. Electrostatic effects occur when this exact balance of positive and negative charges is disturbed, leaving a net charge of one sign on the body - i.e. electrostatic effects occur when *electrical charges are separated.*

### Separation of electric charges

Charge can be separated by various means. Here are some examples.

- Contact of dissimilar materials is followed by physical separation (combing hair).
- The belt of a Van de Graaff generator carries charge. Here also dissimilar materials (the belt and the pulley) are contacted, and separated. Charge is drawn off the belt, and stored on the dome.
- In a battery charge is separated by chemical means.
- Diffusion of charge occurs in living cells. Ions of one sign of charge can move more readily through the permeable wall of a cell, than those of the other sign.
- Charge is separated by convection in thunderclouds.

All charge separation involves the expenditure of energy. The energy is released, for example in a spark, when the charged particles recombine.

## 1-3 FORCES BETWEEN CHARGED BODIES

Energy must be supplied to separate unlike charges because there is an attractive **electrostatic force** between the charges. This electrostatic force is one variety of the general electromagnetic force. It is the basic interaction between charged bodies and it is called electro*static* because the other varieties of electromagnetic force appear only when charges move.

### Properties of the electrostatic force

Experiments show the following.

- Like charges repel. If you connect somebody to an electrostatic generator their hair will stand up because each hair is carries the same sign of charge and the similar charges repel. Likewise, two foam plastic balls fly apart when both are positively charged.
- Unlike charges attract. When opposite charges are transferred to foam plastic balls, the balls are attracted towards each other.
- The magnitude of the electrostatic force between charges increases as their separation decreases.

### Electrostatic force in atomic physics

The electrostatic force binds electrons and nuclei together to form atoms and the same force holds atoms and molecules together in bulk material. The strength of materials is due entirely to electric forces.

The electrostatic force is so potent that only minute deviations from charge equality can be achieved in practice. For example when a 0.2 kg aluminium plate is charged negatively by attaching it to a 250-volt battery, the charge on the plate is about -5 $\mu$C. To make up this huge charge requires that only $10^{13}$ electrons, or 1 for every $10^{12}$ of all the $10^{25}$ electrons on the plate, be added to the plate.

### Electrostatic force in nuclear physics

The strong repulsive electrostatic force between the protons in the nucleus of an atom tends to break up the nucleus, which is held together by the very strong attractive nuclear forces. The collision of a neutron with a uranium-235 nucleus can break the nucleus into two lighter nuclei. These fragments are no longer held together by the short-range nuclear force, but move rapidly apart due to the repulsive electrostatic force between the protons in each nucleus. Thus the release of energy in nuclear fission is mediated by electrostatic forces.

## 1-4 ELECTRIC FIELDS

Electric field is an idea introduced to describe electric forces. In the TV lecture the dome of a charged Van de Graaff generator attracts charged droplets of water. This situation can be described by saying that the charge on the dome of the generator sets up an intense **electric field**, and the charged drops ‘feel’ a force when they are in the space occupied by that field.

A field is something that is defined at all points in a region of space. A familiar example is the gravitational field of the earth, which is often called the acceleration due to gravity ($g$). Another example is the velocity of water in a river which has different magnitudes and directions at different places. Examples to be considered in this unit are electric field, electrostatic potential and magnetic field. An electric field is said to exist at a point in space if a charged particle placed at that point experiences a force that would not be felt by an uncharged particle. We have already seen that a possible explanation for such a force could be the presence of another charged body - but that is not the only thing that can create an electric field. Instead of describing the interaction of charged particles directly in terms of forces it is more fruitful to use a new kind of description involving electric fields. The two descriptions are visualised and compared in figure 1.1.

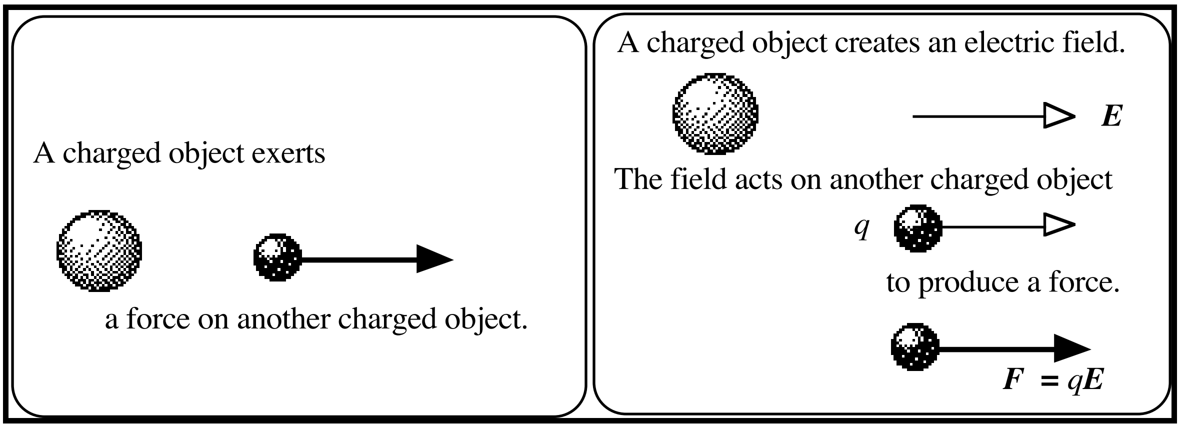


**Figure 1.1 The concept of electric field**
The diagram on the left represents the description of an interaction in terms of forces. On the right we separate the interaction into two parts.

The value of an electric field $\mathbf{E}$ can be defined in terms of the force $\mathbf{F}$ experienced by a particle with a small charge $q$:

$$\mathbf{F} = q\mathbf{E}\,. \qquad \ldots (1.1)$$

Both $\mathbf{F}$ and $\mathbf{E}$ are vector quantities - they have both magnitude and direction.* In this definition the magnitude of the force is equal to $qE$ and the direction of the field is equal to the direction of the force on a positively charged particle. The force on a negative particle is opposite to the field direction. We say that an electric field is uniform when neither its magnitude nor its direction changes from one point to another.

The SI unit of electric field is called the volt per metre (symbol V.m$^{-1}$) which is equivalent to a newton per coulomb (N.C$^{-1}$).

## 1-5 FIELD LINES

A neat way of mapping a two-dimensional slice through a magnetic field is to float some grass seeds or some other kinds of small elongated objects on the surface of a liquid. The electric field causes the seeds to align themselves with the field's direction. An equivalent way of visualising a field in three-dimensions would be to imagine lots of tiny arrows, each one showing the direction of the field at the chosen place. There is, however, another way of visualising electric fields (and other vector fields) - **field lines**. Field lines are continuous directed lines drawn so that at any point on a line the direction of the line (its tangent) shows the direction of the field. Field line diagrams are also three-dimensional so it is not always possible to represent them accurately on flat diagrams. Figures 1.2 and 1.4 are examples where we can draw a two-dimensional picture because they represent situations which have the same configuration in all cross-sections.

Electrostatic field lines start on positively charged surfaces and end on negative charge (figures 1.2, 1.4). If you have an ‘isolated’ charged body the field lines go out to ‘infinity’ as though they are looking for the other sign of the separated charge (figure 1.8).

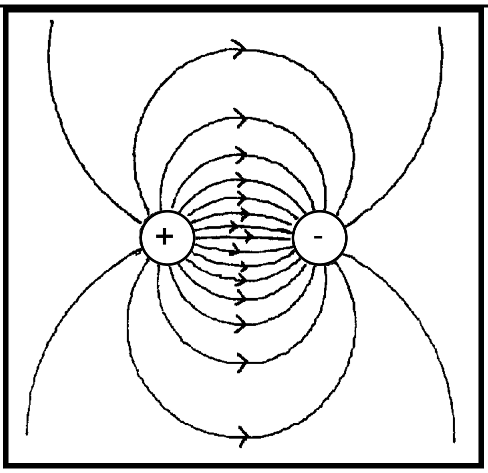


**Figure 1.2 Field lines for two oppositely charged parallel cylinders**

Not only do field line diagrams represent the field direction in a visually obvious way; the spacing of the lines also represents the magnitude of the field. The field is strongest where the lines are closest together.

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* In printed material vectors are indicated by bold-face symbols, such as $\mathbf{E}$. In manuscript we usually put a tilde (~) under the symbol (e.g. $\underset{\sim}{E}$) or an arrow above it (e.g. $\vec{E}$). The magnitude of a vector is indicated by using the same symbol without bold-face or by omitting the tilde or the arrow. Occasionally a component of a vector is indicated in the same way as a magnitude; the context usually makes it clear which is meant.

## 1-6 ELECTRIC FIELD AND CHARGE

### Conductors with static charges

An electrical conductor is an object through which electrons or ions can move about relatively freely. Metals make good conductors. If a net charge is placed on a conductor and it is then left alone, the charge very quickly settles down to an equilibrium distribution. There are several interesting things to note about that situation. (See figure 1.3.)

- The net charge is spread out over the surface of the conductor, but not uniformly.
- There is an electric field in the space around the conductor but not inside it.
- At points just outside the surface of the conductor, the electric field and the electric field lines are perpendicular to the surface.

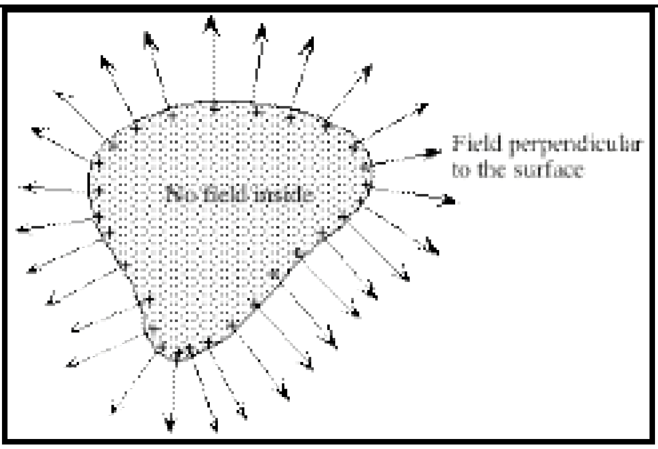


**Figure 1.3 Electrostatic field near the surface of a conductor**
The charge density varies over the surface. The field near the surface is strongest where the charge density is highest.

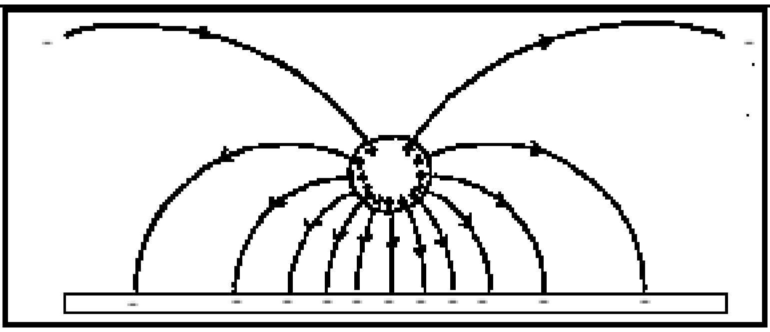


**Figure 1.4 Electric field between a positively charged cylinder and a nearby plane surface.**

We can describe the way that charge spreads out on the surface by specifying the concentration of charge or **surface charge density** - the charge per surface area. The magnitude ($E$) of the electric field at the surface of a conductor is proportional to the surface charge density ($\sigma$). This relation is usually written as

$$E = \frac{\sigma}{\varepsilon} \; . \qquad \text{... (1.2)}$$

The constant $\varepsilon$ is a property of the medium that surrounds the conductor, and is called the permittivity of the medium. If the surrounding space is empty (a vacuum) we indicate that by putting $\varepsilon = \varepsilon_0$, and we call the constant $\varepsilon_0$ (pronounced ‘epsilon nought’) the **permittivity of free space**. Although a vacuum is a pretty rare thing, the permittivity of air is very nearly equal to $\varepsilon_0$, so in electrostatics we usually regard air as being equivalent to empty space.

The SI unit of surface charge density is the coulomb per square metre ($C.m^{-2}$). The SI unit of permittivity is called the farad per metre, $F.m^{-1}$ which, in terms of units that you know already, is equivalent to $C^2.N^{-1}.m^{-2}$. The permittivity of free space is one of the fundamental constants of nature; its value is

$$\varepsilon_0 = 8.85 \times 10^{-12} F.m^{-1}.$$

Equation 1.2 (with $\varepsilon = \varepsilon_0$) is *always* true for conductors in a vacuum. It is a consequence of **Gauss's law**, which is one of the four basic laws of electricity.

### Field lines and electric field strength

Field line diagrams represent the strength of the field at each point provided that they are correctly drawn. In a correctly drawn field-line diagram each line must begin (and end) on equal amounts of charge. The field lines are also drawn so that the field strength is proportional to the concentration of the lines (figure 1.5).

Think of a small imaginary surface, area $A$, drawn perpendicular to the field at some place in space. Count the number, $n$, of field lines that go through the surface. Then the magnitude of the electric field averaged across the area $A$ is represented by the number of field lines per area.

$$E \quad \frac{n}{A} . \qquad ... (1.3)$$

Thus, as the field lines spread out from a charged body the electric field strength falls off correspondingly.

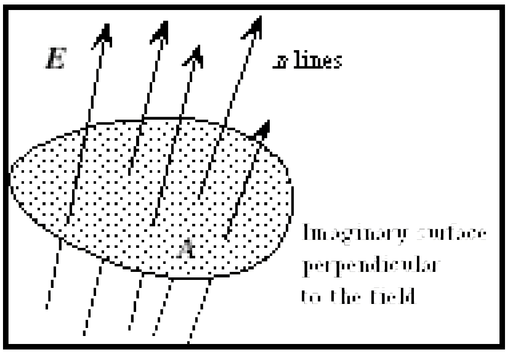


**Figure 1.5 Field strength and concentration of field lines**

The product of the component of the average field perpendicular to the surface and the surface area is called the **flux** of the field through the surface. So the number of field lines through a surface represents the flux of the electric field through that surface.

In principle once a charge distribution is known the electric field can be deduced and vice versa. In practice such calculations can usually be done only numerically (by computer) except for simple geometries. Calculations of the relationship between electric field and charge distribution are vital in many branches of science and technology. Examples range from the study of ion diffusion in living cells, through the design of electrical equipment, to the investigation of thunderstorms.

For highly symmetrical conductors accurate field-line diagrams are fairly easy to draw. Some examples are given in figures 1.6, 1.7 and 1.8.

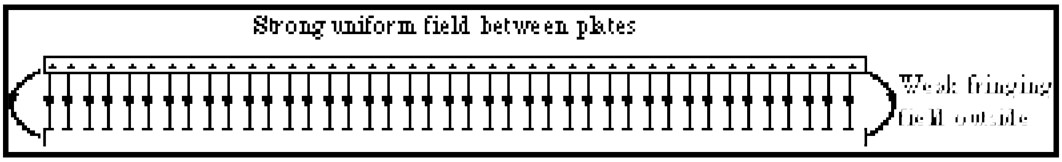


**Figure 1.6 Field lines for parallel plates**

The fringing field at the ends of the plates can be ignored if the plate area is large, or if the plates are close together.

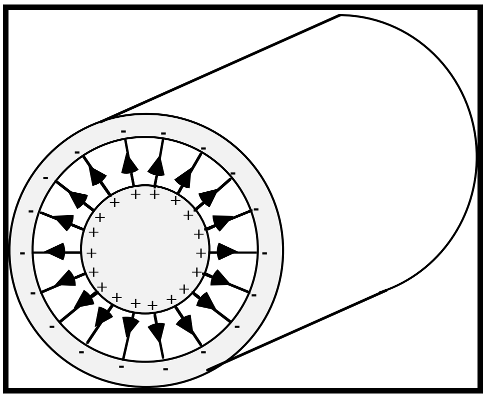


**Figure 1.7 Coaxial cylinders**
The field lines are radial.

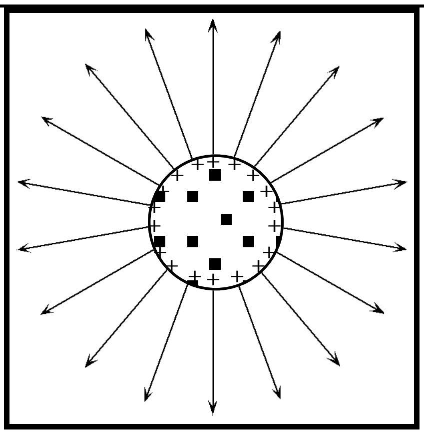


**Figure 1.8 Charged metal sphere**
The field lines are radial in three dimensions. This diagram shows a two-dimensional slice through the centre of the sphere.

## 1-7 ELECTROSTATICS IN INDUSTRY AND COMMERCE

### Electrostatic spray painting

If a surface to be coated is charged, say negatively, then the corresponding electric field will be directed inwards towards the surface. Positively charged droplets of paint from the sprayer are then driven towards the surface by the forces that they experience in this electric field (or, if you like, the positively charged droplets are attracted to the negatively charged surface).

### Electrostatic precipitators

In electrostatic dust precipitators a large negative charge is maintained on spiral wire electrodes, which are arranged between positively charged plates. Negative ions ($O_2^-$, $N_2^-$ etc.) are formed at the wire and the exhaust gases pass between the wires and the plates. The negative ions collide with the dust particles, giving them a negative charge. This negatively charged dust is then driven by the electrostatic field to the positive plate, to which it adheres until collected. Electrostatic precipitators are routinely used in coal-fired power stations; they remove over 95% of the light fly ash from the exhaust gases.

### Xerography

In a photocopier an image of the document is formed on a charged plate of selenium - a photoconductive material. Light reflected from the white areas of the document discharges the illuminated areas of the plate, leaving a charge-image of the document. Charged particles of black printing ink are then attracted to the charged image. Finally, the ink is transferred by direct contact to the sheet of oppositely charged copy paper.

## 1-8 ELECTRICAL SHIELDING

Michael Faraday discovered that inside a closed metal room no electrical effects from outside influences could be detected, no matter how violent the electrical activities outside. As he put it:

> 'I went into the cube and lived in it, and using lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence upon them ... though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every part of its surface.'
>
> [From M. Faraday (1849), *Experimental Researches in Electricity* para. 1174.]

**Demonstration**

Faraday's demonstration is repeated in the TV lecture where one lecturer stands inside a wire mesh cage. A potential difference of 40 kV is established between plates on each side of the cage; the cage rises to a voltage of about 20 kV. Despite the intense electric fields that exist outside the cage, the lecturer is quite safe inside the cage. Indeed, no electric field can be detected inside.

When a hollow conductor is placed in an electric field, charge is redistributed very rapidly over the surface so that the field lines end on surface charges (figure 1.9).

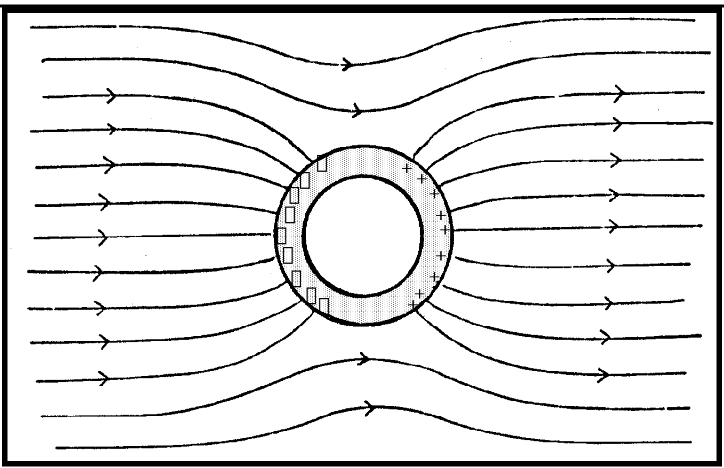
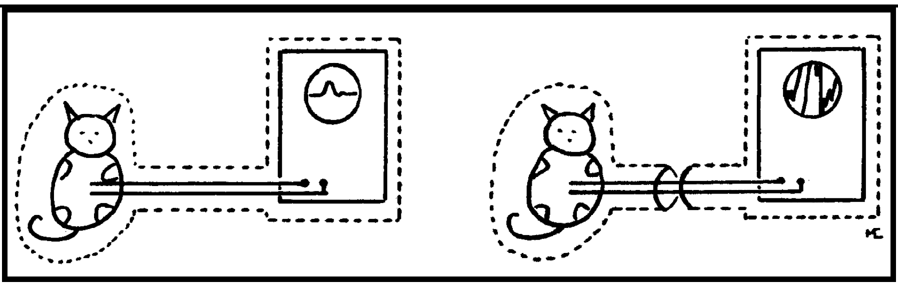


**Figure 1.9 Surface charges on a hollow conductor in an electric field**

There is no field within the conducting walls, and hence there is no field inside the hollow space. Of course charge, *inside* the hollow conductor, can be separated to give an electric field inside the conductor. The point is that no *external* electric field can influence the electrical conditions *inside* the conductor. The inside of the conductor is shielded from outside influences. In practice shielding is never absolutely effective, but it can reach 99.99% effectiveness even with mesh rather than continuous metal. Attention must be paid to the integrity of joins, to doors and to the entry of cables. In the TV lecture, it is shown that the field inside the cage, as measured by the field mill, is hundreds of times less inside the cage than outside. Note that a battery operated field mill was used to avoid the problems of bringing in power leads while maintaining complete shielding.

For successful shielding of experimental apparatus, the whole apparatus including signal and power leads, must all be totally within the same shielded space. Provided that the shield encloses the entire experiment it may be any shape. A mesh shield will also protect apparatus from changing fields of electromagnetic radiation, as long as the mesh size is very much smaller than the wavelength of the radiation. In the TV lecture, it is demonstrated that the mesh cage effectively screens broadcast band radio signals (about 300 m wavelength), but not light (500 nm).

A screened room is a normal feature of any electrophysiology laboratory, where tiny voltages are measured. In other situations screening may be necessary because the external fields are so strong. In plasma physics experiments, for example, huge electrical energy is used to form the plasma so the apparatus which makes sensitive measurements of the electrical properties of the plasma must be thoroughly shielded.



Correctly screened Incorrectly screened

**Figure 1.10 Screening an experiment**

If the screen is in two or more disconnected parts, it is ineffective.

## POST-LECTURE

## 1-9 THE FUNDAMENTAL LAWS OF ELECTROSTATICS

Traditional courses on electrostatics start with the inverse square law for the force between two point charges $q_1$ and $q_2$, Coulomb's law:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \text{.... (1.4)}$$

where $r$ is the distance between the charges, and the force acts along a line joining the charges.

The corresponding expression which gives the magnitude of the radial electric field of a point charge, $q$, (or of a uniformly charged sphere) is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{....(1.5)}$$

where $r$ is the distance from the charge (or the centre of the sphere) and the electric field is in a radial direction.

While this law is of fundamental importance in physics, it is not used in this form either in the later development of electromagnetism, or in the life sciences. Consequently we have based this course on a much more useful law which is equivalent to Coulomb's law: **Gauss's law.** Although we have not formally stated Gauss's law (it is rather abstract mathematically) we have used two aspects of it.

i) The field near a conducting surface is proportional to the surface charge density (eqn 1.2).

ii) At any place the electric field strength is proportional to the areal density of field lines (eqn 1.3).

### Field Lines and Gauss's law

Like many very basic physical laws, Gauss's law appears in many guises, which may be very mathematical. The formal statement of Gauss's law is: 'The total flux of electric field through any simple connected closed surface is equal to the total charge enclosed by that surface, divided by $\varepsilon_0$.' (Flux is a product of field strength and area of an imaginary surface drawn perpendicular to the field.)

We can also express Gauss's law in terms of field lines: 'The total number of field lines passing outward through any closed surface is proportional to the total charge enclosed within the surface.' This statement implies that field lines originate on charges. Field lines passing *out* of the surface *add* to the number of lines; those passing *into* the surface *subtract* from the number of lines.

### Superposition

We have assumed, correctly, that electric fields are additive. Physical quantities which behave additively, like fields, are said to obey the **principle of superposition**. This is not a trivial statement because there are some quantities, such as intensity of coherent waves and the effects of drug dosages, which do not behave additively. Physical quantities which do obey the superposition principle include force, mass, energy, electric field, magnetic field and electrostatic potential.

### Electric dipoles

A pair of oppositely charged particles is often called a **dipole**. The electric field produced by a dipole is like that shown in figure 1.2. One way of describing the properties of a dipole is to specify its **dipole moment**, which is defined as the product of the magnitude of the charge and the separation between the particles: $p = qd$.

## 1-10 ELECTROSTATIC DEFLECTION

A good example of the force experienced by a charge in an electric field is provided by the technique of electrostatic deflection. Consider an electron of mass *m* and horizontal velocity ***v*** entering a region of vertical (upward) electric field ***E*** in a vacuum (figure 1.11).

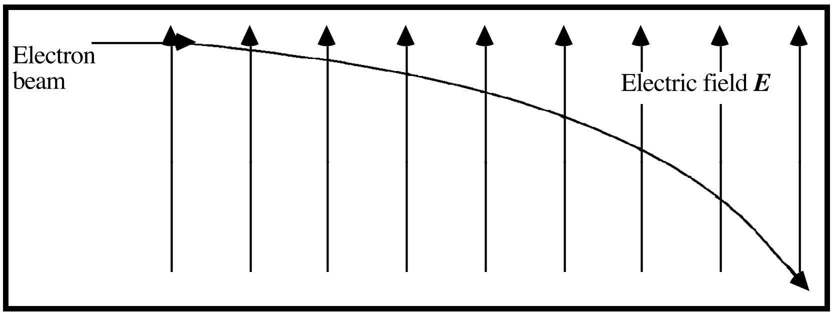


**Figure 1.11 Deflection of an electron beam by an electric field**

The electron will undergo an acceleration of magnitude *eE/m*. Since the electron's charge is negative the direction of this acceleration is opposite to the direction of the field. The component of velocity parallel to the field increases in magnitude while the component perpendicular to the field remains constant. The result is a motion in which the path bends rapidly toward the (negative) field direction. The path is like that of a projectile which is thrown with an initial horizontal velocity and bent downwards by the earth's gravitational field.

**Note**

To analyse the motion, use two components. Displacement component $x$ is perpendicular to the field, to the right in figure 1.11, and component $y$ is parallel to the field lines (up). Since a typical electrostatic force is so strong compared with gravity, we leave gravity out of the analysis.

The $x$-component of the motion is force free, so the component of velocity $v_x$ maintains a constant value $v$ and the horizontal displacement in a time interval $t$ is

$$x = vt.$$

The component of motion parallel to the field is affected by a constant force component $F_y = - eE$. This force produces a constant acceleration component $a_y$ given by the equation of motion, $F_y = m a_y$. To find the velocity component, integrate (find the area under) the acceleration-time graph. That gives

$$v_y = a_y t$$

Then you draw another graph, the velocity time graph, to find $y$. The result is

$$y = -\frac{1}{2}\left(\frac{eE}{m}\right)t^2$$

To find an equation for the shape of the path, eliminate the time from the equations for $x$ and $y$.

$$y = -\frac{1}{2}\left(\frac{eE}{m}\right)E\,\frac{1}{v^2}\,x^2$$

The electron thus follows a parabolic path in the field.

**Demonstration**

The path is demonstrated in the next TV lecture, E2. A special electron beam tube has a longitudinal phosphorescent screen which reveals the path of the ribbon-like electron beam. The vertical ***E*** field is produced by putting a high potential between a pair of horizontal plates inside the tube.

Electrostatic deflection of electron beams is used in cathode ray oscilloscope tubes to make electrical measurements. Amplified signals are applied to horizontal and vertical deflector plates inside the tube, both to provide the time base sweep, and to display the signal under investigation.

## 1-11 QUESTIONS

**Q1.1** Why do you think it took people so long to formulate the notion of electric charge, whereas mass has been recognised since antiquity?

**Q1.2** If you pass a comb through your hair and it acquires a charge of -0.1 $\mu$C, how many excess electrons does it carry? What conclusions can you reach by applying the law of conservation of charge to this situation?

**Q1.3** Which of the following effects are fundamentally electric in nature? Discuss each briefly.

1. tension in a spring,
2. gas pressure,
3. ‘crackles’ from some very dry garments when you undress,
4. ‘crackles’ from treading on dry leaves,
5. gravity,
6. nerve conduction,
7. nuclear fission,
8. the aurora,
9. radioactive □ decay,
10. centrifugal force,
11. the spiral structure of galaxies,
12. moment of inertia,
13. ‘magnetic’ photo albums.

**Q1.4** A charged object, such as the dome of a Van de Graaff generator, is discharged by momentarily touching it with an earth wire. Draw diagrams of the charge and fields involved, and explain what is going on here.

**Q1.5** A spray-painter nozzle normally produces droplets of paint in a cone shaped beam of width 30°. What do you think would be the effect on the beam if each droplet were given a large positive charge as it left the nozzle? Give your reasons. [Hint: remember that the separation of the droplets increases as they move away from the nozzle.]

**Q1.6** A petrol fire at a filling station was allegedly caused by ‘static electricity’. Explain what this claim means, and outline the investigation that a forensic scientist might make to establish whether or not static electricity could have been the cause of the fire.

**Q1.7** A cell membrane has a surface charge density of $-2.5 \times 10^{-6}$ C.m$^{-2}$ on the inside surface and $+2.5 \times 10^{-6}$ C.m$^{-2}$ on the outside. The thickness of the membrane is much less than the size of the cell. Ignore any dielectric polarisation effects.

a) Sketch the electric field between these charge layers.

b) Calculate the magnitude of the electric field
   (i) within the cell membrane,
   (ii) inside the body of the cell.
   This problem, and other similar problems, will be discussed in later sections of this course.

**Q1.8** In the previous problem, what electric force would be experienced by a singly ionised ion of sugar? Compare this with the gravitational force on the same ion.

Data required:

Relative molecular mass of sugar ion = 343

Electronic charge = $1.6 \times 10^{-19}$ C

Mass of proton = $1.67 \times 10^{-27}$ kg

Magnitude of gravitational field = 9.8 m.s$^{-2}$

**Q1.9** a) Sketch the electric field between two concentric cylinders of radii $r_1$ and $r_2$ and length $h$. The charges $+Q$ and $-Q$ on these cylinders are equal in magnitude, but opposite in sign.

b) Obtain an expression for the magnitude of the electric field at a point between these cylinders, at a distance $r$ from their common axis.

**Q1.10** A 2.0 m long cylinder, 0.50 m in diameter, has a charge of $2.0 \times 10^{-3}$ C uniformly distributed over its curved surface.

a) Calculate the electric field at the curved surface of the cylinder.

b) What is the electric field 0.75 m from the axis of the cylinder?

**Q1.11** An experiment is planned which involves the measurement of very small potentials from living fish (in water). Indicate how the requirements for thorough electrostatic shielding might influence the choice of equipment and the configuration of the apparatus.

**Q1.12** A one-cent coin and a twenty-cent coin are placed about 10 cm apart on a sheet of glass. The 1 c coin bears a charge of $+Q$ and the other coin, $-Q$. Sketch the field lines in the vicinity of the coins. Where will the electric field be strongest?

# E2 ELECTRIC POTENTIAL

## OBJECTIVES□

### Aims

The most important new concept that you should aim to understand from this chapter is the idea of potential difference. Associated with this you will learn how to use the concept of energy in the study of electrostatic phenomena. You will also learn the essential physics involved in the electrical behaviour of non-conducting materials, called dielectrics, using the concepts (which have already been introduced) of charge, charge density and electric field.

### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

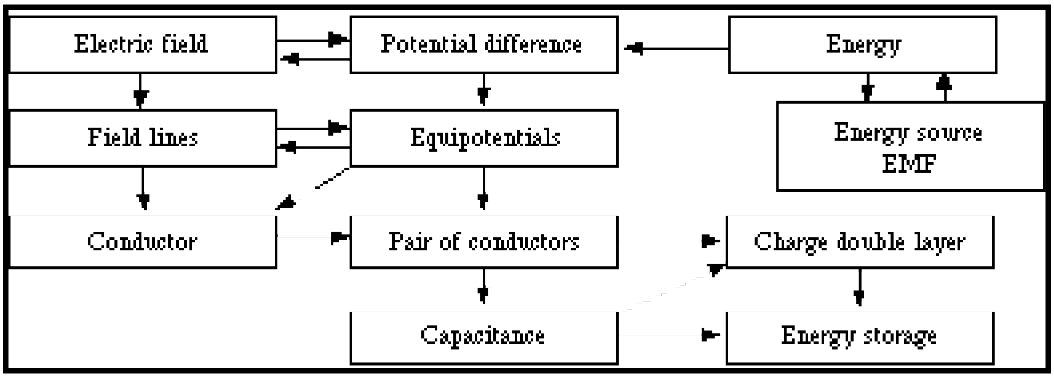
1. Explain, interpret and use the terms:

   *induced charge, electric potential, voltage, potential difference, electromotive force, volt, charge double layer, potential discontinuity, capacitance, capacitor, dielectric, relative permittivity, equipotential, equipotential surface, potential gradient.*
2. Give examples of the magnitudes of typical voltages encountered in scientific work and everyday life.
3. State and apply the relations among potential difference, surface charge density, charge, surface area, relative permittivity and separation distance for parallel plate capacitors and charge double layers.
4. State and apply the definition of capacitance. Describe and explain the effect of dielectric materials on capacitance.
5. State and apply expressions for the energy stored in a capacitor.
6. State and apply the relation between potential difference and electric field strength for uniform fields.
7. Apply energy conservation to the motion of charged particles in electric fields.
8. (a) Interpret equipotential diagrams.

   (b) Sketch equipotentials for simple geometries of charged static conductors.

   (c) Explain why the surfaces of conductors are usually equipotentials.

### CONCEPT DIAGRAM



# PRE-LECTURE□

## 2-1 INTRODUCTION - ELECTRIC POTENTIAL

In chapter E1 you saw that one way of explaining the effect of a static distribution of charges is to describe the electrostatic field ***E*** in the space around the charges. In this chapter we explore another way of doing the same thing - which is to describe the electric potential difference between various points in the space. These two kinds of description are essentially equivalent - if you know one of them you can work out the other. The relation between electrostatic field and potential difference is the same as that between conservative force and potential energy (PE); see chapter FE5. The change in PE is equal to minus the work done by the force. Since electric field is force per charge, potential difference can be defined as a change in potential energy per charge.

One of the advantages in talking about potentials rather than electric fields is that whereas force and electric field are vector quantities, potential energy and potential are scalar quantities. Scalars are more easily calculated and described than vectors.

## 2-2 INDUCED CHARGE

Consider what happens when a body is introduced into a region of electric field, for example by bringing a charged object close to the conducting body. If the body is a metallic conductor some of the electrons in it are free to move. They move in a direction opposite to that of the electric field, leaving a positive charge behind them. We say that charges have been **induced** on the ends of the body by the electric field. Charge separation can also be induced in insulating materials when positive and negative charges within the molecules of the material rearrange themselves, leaving net surface charges of opposite sign on opposite sides of the conductor. Such **induced charges** are formed when pieces of paper are attracted to a charged comb. See figure 2.1

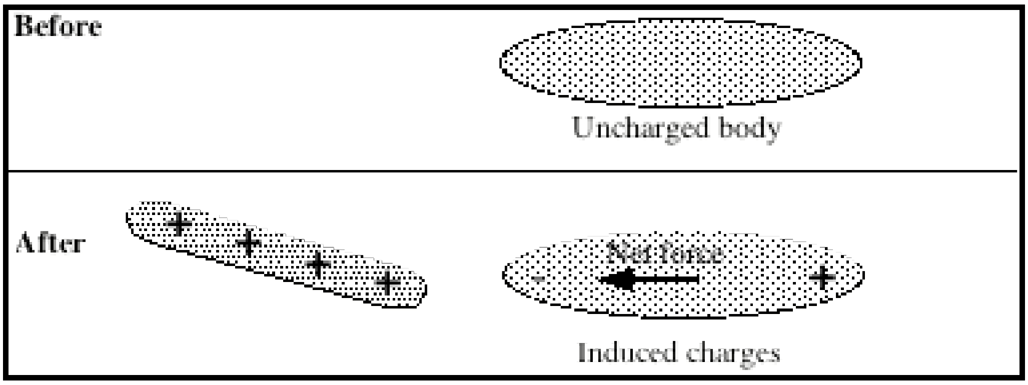


**Figure 2.1 Induced charge**

### *Question*

**Q2.1** The charges induced on a piece of paper by an external electric field are equal in magnitude and opposite in sign, and yet the paper is still attracted to a comb (figure 2.1). Why?

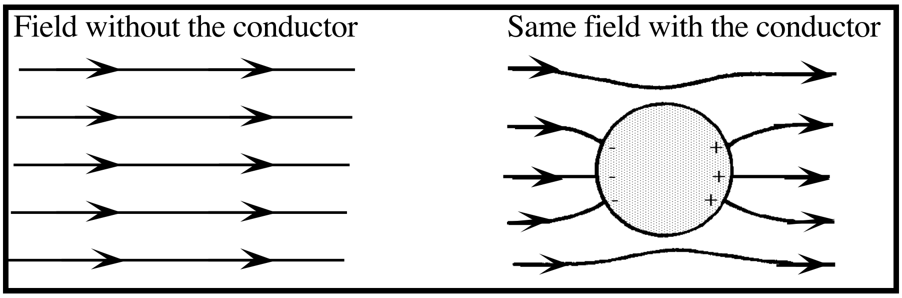


**Figure 2.2 A conducting body in an electric field**

The original field lines are distorted by these separated charges (figure 2.2). Some of them terminate at the surface on one side of the body, and new field lines originate from the other side. If the body is elongated the separation between the charges can be quite large i.e. the charges act, effectively, as isolated electric charges.

## LECTURE

## 2-3 ELECTRIC POTENTIAL

Any charged particle located in a region of electrostatic field experiences a force. The force on the particle at any place is determined by the particle's charge and the value of the field: $\boldsymbol{F} = q\boldsymbol{E}$. If the particle moves from one place to another within that region, the electrostatic force does work on the particle and its potential energy changes. Just as we can associate an electric field with each point in the space, we can also define an electrical potential $V$ for every point, such that a charged particle's potential energy $U$ at the point is given by the simple relation

$$U = qV. \qquad \text{... (2.1)}$$

Since potential is defined at each point in space it is a field, but unlike the electric field $\boldsymbol{E}$, it is a scalar so it has no direction. Since potential energies are always reckoned from an arbitrary zero level, so is potential. It is usually more meaningful to talk about the potential difference ($\Delta V$) between two points in space than the potential ($V$) at one point.

The colloquial term ‘**voltage**’ can refer to one of two distinct physical ideas - EMF or potential difference. **EMF** is associated with sources of electrical energy while **potential difference** is created by distributions of charge. (The term EMF is derived from electromotive force, a name that is no longer used, because the quantity is not a force.)

### Electric potential and sources of electric energy

Sources of electrical energy such as batteries and sources of electrical signals such as nerve cells can separate electric charges by doing work on them. This energy put in to a system of charges can be described in terms of the ‘voltage’ or **EMF** of the source. For sources of electricity:

$$\text{EMF} = \frac{\text{energy supplied by source}}{\text{charge passed through source}} \,.$$

The separated charges produce an electrostatic field and a potential at every point in the space around them.

When a source of EMF is not connected to an external circuit, the potential difference between its terminals becomes equal to the EMF of the source, but while charges are actually flowing through the source energy dissipation within the source may cause the potential difference to be less than the EMF. This point is explored in more detail in your laboratory work.

The SI unit of both potential difference and EMF is the volt; (symbol V) defined as one joule per coulomb: $1\ \text{V} \equiv 1\ \text{J.C}^{-1}$. The unit is named after Alessandro Volta who astonished the scientific world in 1800 when he invented the first source of continuous electric current, the first battery.

*Voltage* is probably the most commonly used term in electricity. That is because the most significant thing about any source of electricity or about any electrical signal is its voltage or EMF. Some examples are listed in the following table.

| **Source** | **Voltage (EMF)** |
|---|---|
| EEG signals | ~ 1 $\mu$V |
| ECG signals | ~ 1 mV |
| cell membrane | ~ 100 mV |
| flashlight cell | 1.5 V |
| domestic electricity | 240 V (Australia, U.K.)<br>115 V (U.S.A., Canada) |
| Van de Graaff generator | $10^5$ V |
| power transmission | $3.3 \times 10^5$ V |
| thunderstorm | > $10^8$ V |

## 2-4 ELECTRIC FIELD AND POTENTIAL

You have already seen that electric field is force per charge and potential is potential energy per charge. So electric field and potential are linked in the same way that the concepts of force and potential energy are linked. You will recall, from chapter FE5, that change in potential energy is defined as the negative of the work done by a conservative force, $\Delta U = - \Delta W$. You should also remember that to calculate work done by a given force, we integrate the force with respect to displacement or, equivalently, find the area under a force-displacement graph. So the way to calculate potential difference from electric field is the same: integrate the field with respect to displacement. It is worth recalling why this straightforward relation works. It is because nature has provided a special class of forces, conservative forces, for which the work done by the force on a particle depends only on the starting and finishing points of its motion. The details of the path between the points don't affect the amount of work done. The electrostatic force happens to be one of those conservative forces, so the work that it does on a particle, and hence also the work per charge, depends only on the starting and finishing points. That is why potential can be defined uniquely at each point in space.

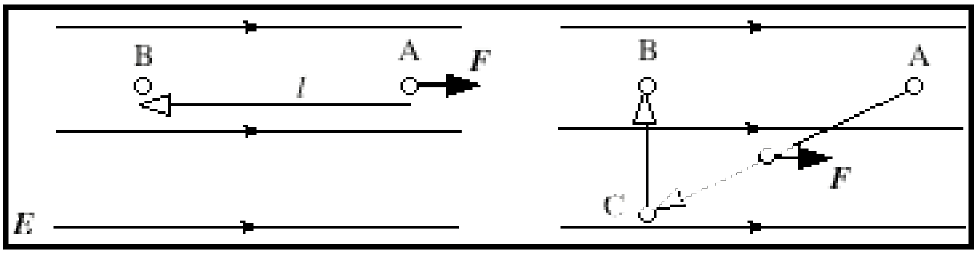


**Figure 2.3 Work done by a uniform field**
The total work done by the electrostatic force is the same in both cases.

In a course like this we don't do complex mathematical problems, so we need consider only a simple but surprisingly useful example: the case of a uniform electric field (figure 2.3). Suppose that we have a particle, charge $q$, which goes from A to B as shown in the left hand part of figure 2.3. (An external force would be needed to achieve that, but we are not really interested in that aspect.) The size of the electrostatic force on the particle is equal to $qE$ and it is directed to the right. Since the field is uniform the force doesn't change and the work done by the electrostatic force is just the product of the displacement and force's component in the direction of the displacement (which is negative in this example).

$$W = -qEl\,.$$

Since the change in PE is equal to minus the work done by the electrostatic force

$$U_B - U_A = qEl\;.$$

Finally the potential difference between points A and B is the change in PE per charge, i.e.

$$V_B - V_A = El\;. \qquad \text{... (2.2)}$$

In this example point B is at a higher potential than A. One way of seeing that is to note that you, an external agent would have to do work on a positively charged particle in order to push it from A to B against the electrostatic force.

Now suppose that instead of going straight from A to B, the particle went via point C, as shown in the right-hand part of figure 2.3. On the diagonal part of the path, the magnitude of the force component is less but the displacement is longer. It's actually quite easy to show that the work is the same as for the path AB. On the second stretch, from C to B, the component of the force in the displacement direction is zero, so there is no more work. The work done over the paths AB and ACB is identical. In fact it is the same for all possible paths from A to B. In this example we have actually shown that the potential at C is the same as that at B. Furthermore it's the same at all points on the line BC. We can call the line BC an **equipotential**.

The relation between potential and field (equation 2.2) can be looked at in another way. We can express the component of the electric field in terms of the change in potential and the displacement.

For a uniform field: $$E_x = -\frac{\Delta V}{\Delta x}\;;$$

or more generally $$E_x = -\frac{\mathrm{d}V}{\mathrm{d}x}\;. \qquad \text{...(2.2a)}$$

This relation says that the $x$-component of the electric field is equal to the negative of the gradient (in the $x$-direction) of the potential. (That's not surprising because we got the potential by taking the negative integral of the work per charge.) The sign is important here because we are now talking about a component of the field, rather than its magnitude, and we need to take account of the directions of both field and displacement.

This relation is also the origin of the SI unit name, volt per metre, for electric field strength.

## 2-5 CONDUCTORS

### Conductors as equipotentials

Every part of a conductor (which is not carrying a current) is at the same electric potential. The conductor is an **equipotential region** and its surface is an **equipotential surface**. If the distribution of charges near the conductor is changed, the conductor will always remain an equipotential, although the value of that potential depends on the charge on and near the conductor.

To see why conductors are equipotentials, suppose we have a one that is not an equipotential. In that case there would then be an electric field in some part of the conductor and the free charges would immediately be accelerated by the electric field. The result would be a continual redistribution of the charges in the conductor until the internal field was reduced to zero.*

A current-carrying conductor (e.g. a wire) by contrast is not strictly speaking an equipotential; there must be some potential drop along the direction of the current in order to keep the charges flowing. However in many applications, the potential change along the conductors is much less than that across the generator or the load. To that extent even current carrying conductors may be regarded as equipotentials. It is common practice to regard the earth conductors in circuits as

---

* In fact this redistribution occurs extremely rapidly; the fields fall exponentially to zero with a time constant $\tau = \varepsilon/\sigma$ where $\sigma$ is the conductivity of the material (see chapter E3). For copper, as an example, the *relaxation time* $\tau$ is about $10^{-19}$ s. Even for relatively poor conductors like rock or living tissue, the relaxation time is much shorter than 1 $\mu$s.

equipotentials (at zero volts). Sometimes this is not good enough, and one is faced with earth loop problems: spurious signals arise in amplifiers due to neglected currents in, and potential drop along, the earth conductors.

In the design of electron guns in (for example) electron microscopes, explicit use is made of the fact that conductors are equipotentials. Potentials applied to the electrodes of the gun establish the electric field which accelerates the electron beam (see chapter E3).

## Pairs of conductors

For a pair of conductors, each of which is an equipotential, the potential difference is related to the electric field between them. Indeed, either potential or field can be used to specify the electric state of the conductors. A simple example is a pair of plane parallel conductors (figure 2.4)

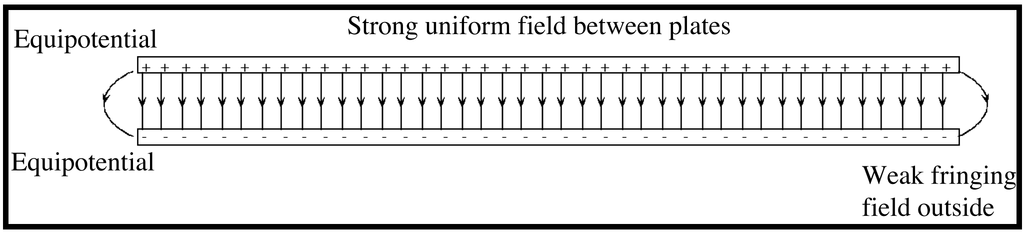


**Figure 2.4 The electric field of charged parallel plates**

When the plates are charged, positive on one and an equal negative charge on the other, the charges spread out uniformly over the inside surfaces of the plates. The electric field, represented by field lines running from the positive to the negative charges, is approximately uniform between the plates. By making the plates sufficiently close, the non-uniform fringing field at the edges becomes relatively unimportant. The field (and field lines) are perpendicular to the plates because the plates are conductors (see chapter E1). The symmetry of the arrangement ensures the uniformity of the field (represented as equally spaced parallel field lines everywhere normal to the surface of the plates).

We have already discussed the connection between field and potential for the case of a uniform field. The electric field between the plates is equal to the potential difference between them divided by their separation. For conductors of arbitrary shape (figure 2.5) the electric field between them is non-uniform.

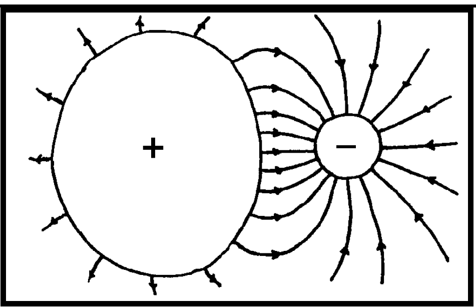


**Figure 2.5 The electric field between two charged conductors of arbitrary shape**

The point to be remembered is that there is a unique electrostatic potential between any pair of conductors.

### Demonstration

In the video lecture, the voltage between two conductors is measured with the voltmeter leads touching the conductors at several different pairs of points. The voltage is shown to be independent of the location of the voltmeter probes.

**Demonstration**

A demonstration of the electrostatic deflection of a beam of electrons which occurs in the video lecture is pertinent to chapter E1. See §1-10.

## 2-6 ELECTRIC DOUBLE LAYERS

Two closely spaced layers of charge, equal in magnitude and opposite in sign, comprise a **charge double layer**. Such double layers occur in the membranes of all living cells. An understanding of their electrical properties is essential in studying the mechanism of nerve transmission and cell metabolism. Here we consider the simplest type of double layer, where the layers of charge are on parallel plane conductors. By supposing that the areas of the two plane conductors are very large, on the scale of their separation, the mathematical description becomes very simple, but the physical ideas involved apply equally well to other shapes and areas of double charge sheets.

Consider two conductors having plane parallel faces a small distance $l$ apart and charged with surface charge density of magnitude $\sigma$. The material in the space between the conductors has permittivity $\varepsilon$ (figure 2.6).

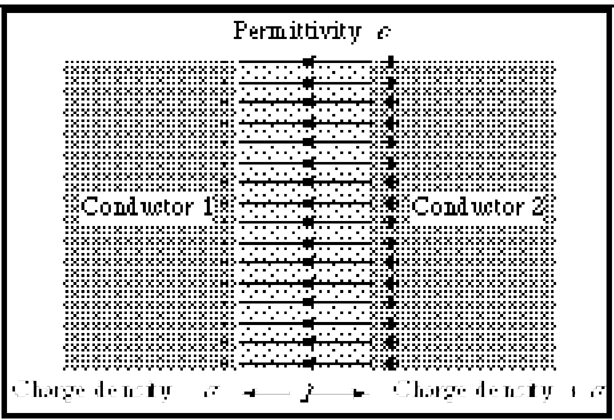


**Figure 2.6 Charge double layer on parallel conductors**

The conductors are sufficiently close so that the effects on the non-uniform field at the edges may be neglected. Then the double layer has the following properties.

- Between the charge layers, the magnitude of the electric field is uniform, directed from the positive towards the negative charges, and of strength (given by equation 1.2)

$$E = \frac{\sigma}{\varepsilon} \qquad \text{....(2.3)}$$

  i.e. the lines of ***E*** terminate on the charges according to Gauss's law.

  The electric field is perpendicular to the conductor surface - can you see why? The magnitude $E$ of the field does not depend on the separation $l$.

- Within the conductors, the electric field is necessarily zero. Each conductor is an equipotential region.
- There is a potential difference between the two conductors given by

$$\Delta V = El = \frac{\sigma l}{\varepsilon} \qquad \text{.... (2.4)}$$

  In figure 2.6, conductor 2 is at the higher potential since it is positively charged. For fixed amounts of charge, $\Delta V$ increases as the spacing $l$ increases.

- Often the spacing $l$ is very small, and we are not very concerned with the region inside the double layer. In effect the charge double layer represents a *potential discontinuity* between the two conductors. In fact, whenever a potential difference exists between two contiguous pieces of matter a charge double layer is involved. Examples include living cells, batteries, thermocouples, semiconductor junctions, etc.

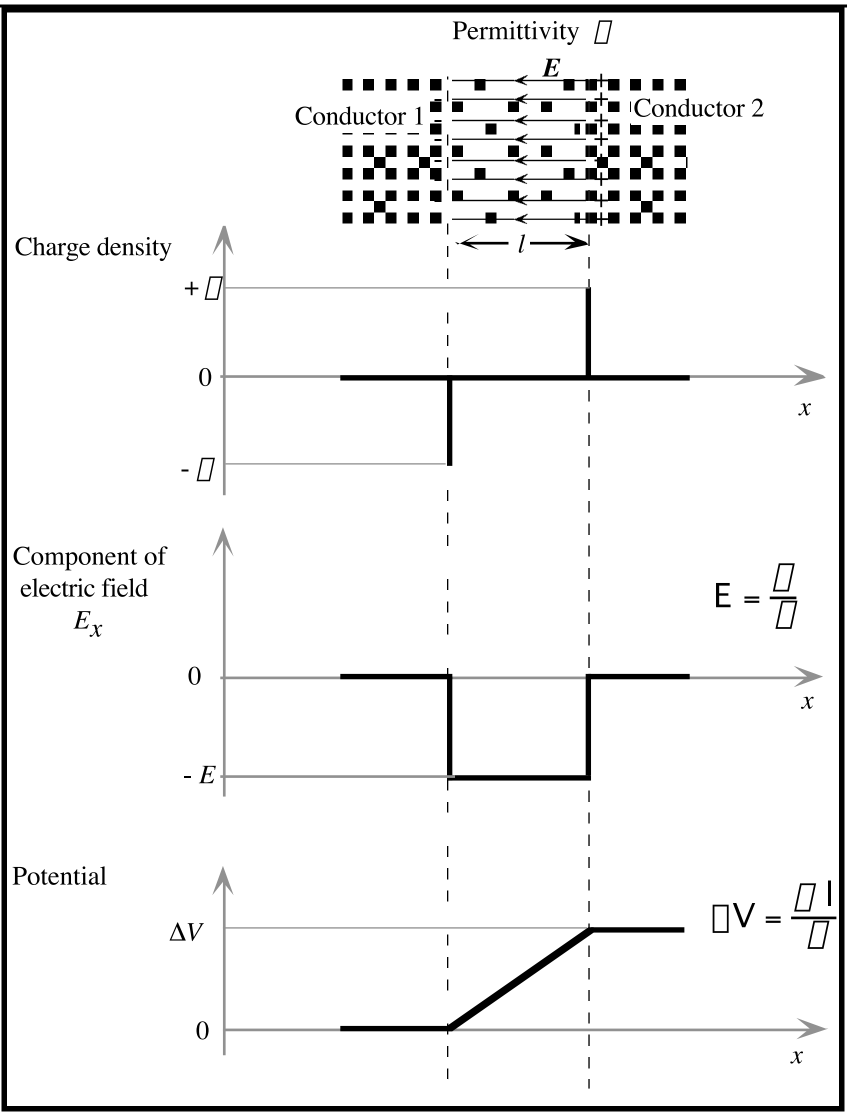


**Figure 2.7 Charge, field and potential in a double layer**
The horizontal axis in all cases represents position.

The properties of a charge double layer are depicted in the graphs of figure 2.7. These diagrams illustrate that a charge layer represents a discontinuity in the electric field of magnitude $\frac{\sigma}{\varepsilon_0}$. A charge double layer represents a discontinuity in the potential, of magnitude $\frac{\sigma l}{\varepsilon_0}$.

**Demonstrations**

Some of the properties of charge double layers are demonstrated in the video lecture.

i) The field mill is mounted with its measuring aperture flush with the surface of an aluminium plate of diameter 0.70 m. A second plate, supported by a high quality insulator is held 0.05 m from the first plate.

The insulated plate is charged by momentarily connecting a 50 V battery between the plates. The field mill meter reads 1 $kV.m^{-1}$ (50 V ÷ 0.05 m). The field between the plates is 1 $kV.m^{-1}$ as the plates are close and edge effects are comparatively small. Thus the magnitude of the surface charge density on the plates is uniform and is related to the electric field by equation 2.3.

The separation of the plates is increased, keeping the charge and the charge density constant. The meter continues to read 1 $kV.m^{-1}$ confirming that *E* is indeed independent of the separation. (The slight fall in *E* is due to the increasing significance of the edge effects as the separation increases.)

ii) The same aluminium plates were placed very close together (about 1 mm) and charged by momentary contact with a 250 V battery. The plates were then drawn apart slowly. An electrostatic voltmeter connected between the plates showed that the potential *rose* as the plates were separated, more or less in proportion to the separation. The potential rose to above 8 kV when the plates were well separated.

This is consistent with equation 2.4 which shows that the voltage increases with separation if the charge is kept constant. In this demonstration the separation $l$ was increased about 30 times, and the voltage rose in essentially the same ratio from 250 V to 8 kV.

## 2-7 CAPACITANCE

### Definition of capacitance

If separated charges $+Q$ and $-Q$ are placed on a pair of conductors, then there will be a potential difference $V$ between the conductors. (Note that here $V$ means the same as $\Delta V$.) Unless the material between the conductors breaks down and starts to conduct the potential difference is found to be proportional to the charge. So the ratio

$$C = \frac{Q}{V} \qquad \text{... (2.5)}$$

is a constant called the **capacitance** of the pair of conductors. The capacitance depends *only* on the shapes of the conductors, their separation and the composition of the insulating material between them. It does not normally depend on the voltage or the charge.

The name capacitance is a modernised version of the old name *capacity* which expresses how much separated charge a capacitor can hold in terms of the voltage. In the old days capacitance used to be measured in units of *jars* because the first capacitors were glass jars coated inside and out with metal foil. The SI unit of capacitance is the coulomb per volt, which has the special name of farad (symbol, F). In most laboratory work we deal with capacitances in the range of 1 pF ($10^{-12}$ F) through 1 nF ($10^{-9}$ F) to 1 $\mu$F ($10^{-6}$ F) and beyond. One microfarad is a decent sized capacitance.

### Parallel plate capacitor

Mathematically the simplest kind of capacitor consists of two identical flat sheets of metal separated by a thin layer of insulating material. When the capacitor is charged we have a charge double layer for which the potential difference is given by equation 2.4:

$$V = \frac{\sigma l}{\varepsilon} .$$

Combining this equation with the definition of surface charge density ($\sigma = Q / A$) and the definition of capacitance (equation 2.5) gives a formula for the capacitance:

$$C = \frac{\varepsilon A}{l} . \qquad \text{.... (2.6)}$$

You can now see where the unit for permittivity, $\mathrm{F.m^{-1}}$, comes from. Just put F for capacitance, $\mathrm{m^2}$ for area and m for separation into equation 2.6.

Equation 2.6 is a special case of the general result that, for any pair of conductors

$$C = \varepsilon \left( \frac{\text{area}}{\text{spacing}} \right) \times F \qquad \text{... (2.7)}$$

where $F$ is a dimensionless factor (a pure number) that depends on the actual shapes and arrangement of the conductors. For the parallel plate capacitor, $F = 1$.

### The effect of a dielectric

It is shown in the video lecture that if a parallel plate capacitor is charged, and then a sheet of perspex is inserted between the plates the voltage across the capacitor falls.

Since for a capacitor $V = \frac{Q}{C}$ and in this case the charge does not change, it follows that the effect of the perspex is to increase the capacitance. The effect is already predicted by equation 2.6 if we say that the permittivity of perspex is larger than that for air. In general, capacitance is increased if an insulating material replaces vacuum between the plates. A material that produces this effect is called a **dielectric**.

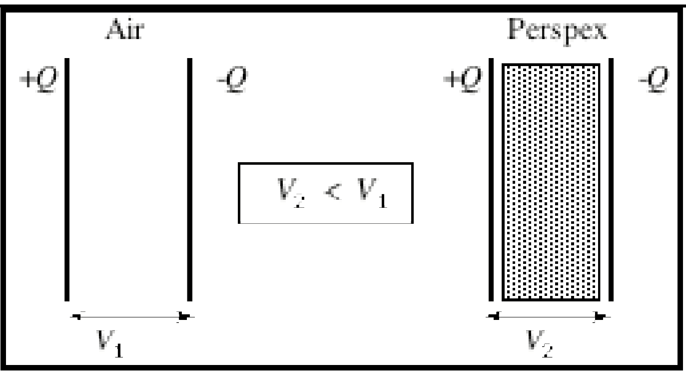


**Figure 2.8 Effect of a dielectric on a charged capacitor**

The effect of the perspex can be understood by considering its molecular structure. The electron clouds are bound to their respective molecules to the extent that they are not free to flow through the material (in contrast with conductors). When the perspex is placed in the charged capacitor, those electron clouds are displaced slightly by the electric field within the capacitor. In the bulk of the material there is no average effect as the charge induced on one end of a molecule is cancelled by the charge of opposite sign induced on the end of the adjacent molecule, but bound **surface charges** appear on the surfaces. Each molecule behaves like a dipole and the dipoles are aligned with the field. Figure 2.9 is a very schematic representation of that effect.

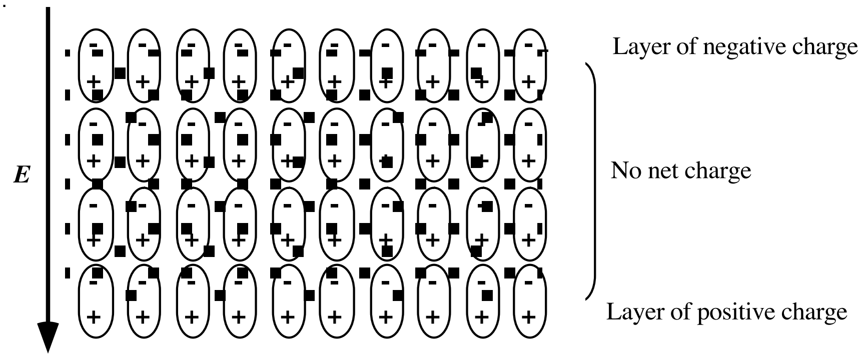


**Figure 2.9 Microscopic picture of an electrically polarised dielectric**

These bound surface charges are opposite in sign to the free charges, ±$Q$, on the metal plates (figure 2.10). They have the effect of *reducing* the total electric field strength within the material, and hence of *reducing* the voltage between the plates.

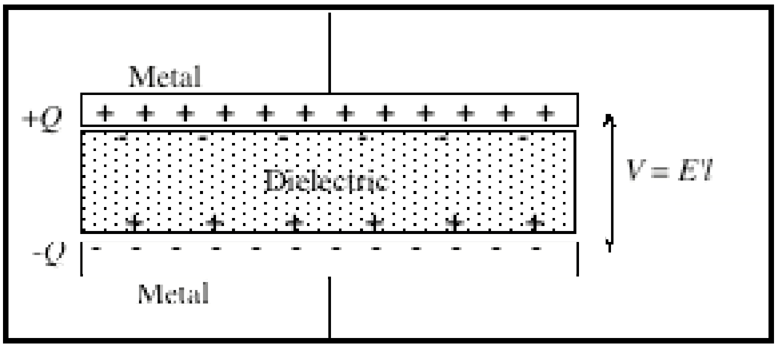


**Figure 2.10 Charges on the capacitor plates and on the dielectric**

Instead of specifying values of permittivity for various dielectrics, it is more meaningful to give a factor by which a dielectric will increase the capacitance above the vacuum value. This factor, which we will call **relative permittivity**, is defined as the ratio of the permittivity of the dielectric to the permittivity of free space:

$$k = \frac{\varepsilon}{\varepsilon_0} \qquad \text{.... (2.8)}$$

Although the term *relative permittivity* describes this quantity precisely, it is more commonly known as **dielectric constant**.

The capacitance of a parallel plate capacitor with a dielectric material between the plates is then

$$C = k\varepsilon_0 \frac{A}{l} \ .$$

For perspex, $k = 2.0$ and for water, $k = 80$.

**Energy stored in a capacitor**

Since work must be done to separate charges, a charged capacitor contains stored electric potential energy:

$$U = \tfrac{1}{2}CV^2 \qquad \text{... (2.9).}$$

**Note**. If you are interested in seeing where this relation comes from, follow this argument. The work done in separating a charge d$q$ when the capacitor voltage is $V$ is

$$\mathrm{d}W = V\mathrm{d}q$$

$$\therefore \ \mathrm{d}W = CV\,\mathrm{d}V \quad [\text{since } \mathrm{d}q = C\mathrm{d}V]$$

$$\therefore \ \int_0^W \mathrm{d}W = \int_0^V CV\,\mathrm{d}V = \tfrac{1}{2}CV^2$$

**Example**. In the capacitor storage bank of the TORTUS tokamak, there is a total of 3.3 farads of capacitance at a voltage of 400 volts. The stored energy ($\frac{1}{2}CV^2$) is a quarter of a million joules.

**Use of capacitance measurements**

Measurements of the capacitance of living cell membranes can be used to deduce the thickness of the membranes, since the capacitance depends inversely on the thickness.

## 2-8 EQUIPOTENTIALS AND ELECTRIC FIELDS

You have already seen that any conductor which is not actually carrying an current is an equipotential. But the ideas of potential and equipotential are not restricted to conductors. The potential difference between *any* two points (whether on conductors or not) is uniquely defined.

Potential differences between conductors can generally be measured by simply connecting a suitable voltmeter. It is much harder to measure potential differences in air, for example. Most instruments so distort the field that it bears little relationship to the situation existing before the measurement was attempted. For example, in fine weather the potential (relative to the ground) at a point 2 m above the ground is about 300 V, due to the permanent positive charge on the upper atmosphere. However if you stand up outside, the potential at your head is zero and the field lines are distorted by your presence.

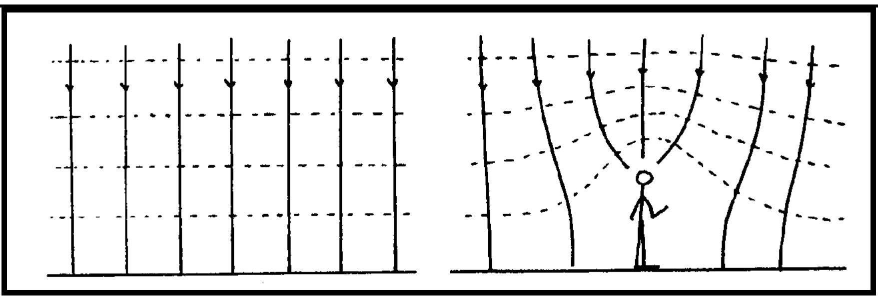


**Figure 2.11 Distortion of the Earth's electric field**

The terrestrial electrostatic field is distorted by a person. The ground and the person together form a continuous conductor which must be an equipotential. The dotted lines represent equipotential surfaces in the air, at intervals of about 100 volts.

Equipotential surfaces are in general imagined (mathematical) surfaces connecting points in vacuum, in air or other material which are at the same potential. Figure 2.12 shows the cross section of a pair of charged wires. The solid lines represent the equipotential surfaces. (This example is shown as a computed animated diagram in the video lecture.) The equipotentials of a dipole are similar to the pattern in figure 2.12.

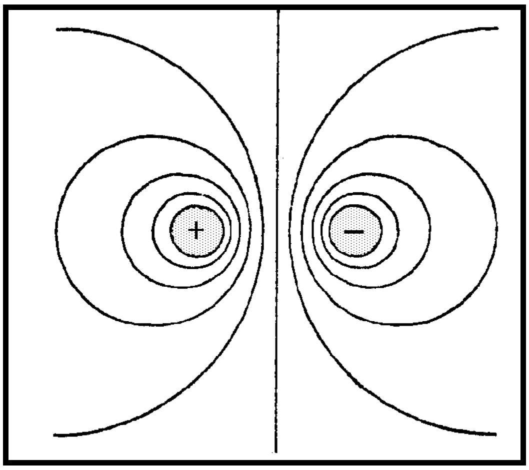


**Figure 2.12 Equipotentials between charged conducting cylinders**

### Equipotentials and field lines

Equipotentials and field lines are closely related. Either is an adequate way of describing the state of electrification of a region. To see the relationship between them, remember that the electric field is just the negative potential gradient.

The strength of the field is measured by the closeness of the equipotentials, and the direction of the field is always at right angles to the equipotential surfaces. Equipotential diagrams resemble contour maps. In a map, the slope is represented by the closeness of the contours, and the direction of slope is perpendicular to the contours.

## POST-LECTURE☐

## 2-9 QUESTIONS

**Q2.2** On a fine day in open country the naturally occurring charge on the ground is -1.00 nC.m$^{-2}$. Calculate the electric field and potential difference (relative to the ground) at a height of 3.00 m.

**Q2.3** By the time you have finished this course you should have some feeling for the order of magnitude of the potential difference involved in various situations. Can you find out the order magnitude of potential difference involved in each of the following?

1. Electrocardiography [*example of answer* ~ 1 mV]
2. Accelerating voltage in a colour TV set
3. Minimum detectable with tongue
4. Minimum felt as shock with two fingers of one hand
5. Minimum capable of causing fatal electric shock (across hand - foot)
6. Used in electrostatic precipitators
7. Motor car ignition
8. Generated in a solar photovoltaic cell
9. Of a wrist-watch batter
10. A piezo-electric gas lighter
11. Induced in a receiving aerial by a transmitter 10 km distant
12. Combing dry hair
13. Corrosion of galvanised iron
14. Nerve transmission
15. Power for electric trains
16. Operating the telephone system
17. In-hand calculators

18 Electric welding
19 Between the mains live and neutral wires
20. Between the mains earth and neutral wires
21. To make a frog's leg twitch
22. Between ground and the atmosphere at a height of 1 km, in fine weather
23. An electric insect zapper
24. Ionisation of an alkali metal, M → $M^+$ + e.

**Q2.4** In a vacuum tube the anode is maintained at a potential of 2.5 kV relative to the cathode. The anode to cathode spacing is 0.10 mm. An electron is released from the cathode and accelerated by the electric field between anode and cathode. What is its kinetic energy on striking the anode? Express your answer

i) in symbols,
ii) in joules.

**Q2.5** A living cell has a potential difference of 90 mV between the interior of the cell and the extra-cellular fluid. The interior is negative with respect to the extra-cellular fluid. What can you deduce about the electric double layer at the cell membrane?

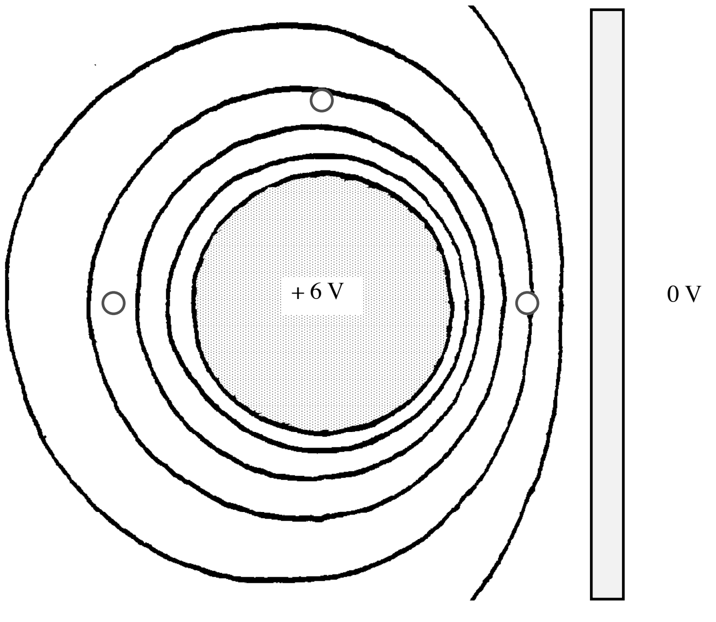
**Q2.6** The membrane of a cell is electrically equivalent to a parallel plate capacitor of area 1.0 mm$^2$ and thickness 1.0 $\mu$m. The relative permittivity of the membrane is 8.0. The voltage across the capacitor is 90 mV. Find:

i) the capacitance,
ii) the magnitude of the electric field in the membrane,
iii) the stored electrical energy,
iv) the magnitude of the separated charge.

**Q2.7** The electric fish *Gymnarchus niloticus* maintains a formidable charge on its head and a charge equal in magnitude and opposite in sign on its tail.

Sketch the general form of equipotentials that you would expect for the fish swimming in deep water (as seen from the side view). How would the shape of the equipotentials change if the fish approached the surface of a metal ship (i.e. a very good conductor)?

**Q2.8** A spherical conductor is maintained at a potential of +6 V relative to an earthed conducting plane. The diagram shows the plane, the spherical conductor drawn to actual size and equipotentials drawn at 1 V intervals.



i) Sketch in the field lines. About 10 lines would be suitable.
ii) By taking measurements from the diagram, evaluate the electric field magnitude and direction) at points 10 mm to the right of the sphere, 10 mm above the sphere and 10 mm to the left of the sphere [these points are indicated by small dotted circles].

## 2-10 APPENDIX - MORE ON ELECTRIC DOUBLE LAYERS

In the lecture, the discussion of electric double layers was restricted to conducting planes. Now we consider them more generally: just two sheets of charge. As before they are uniform and parallel. But now the means of support is not specified.

First consider a single sheet of positive charge (figure 2.13).

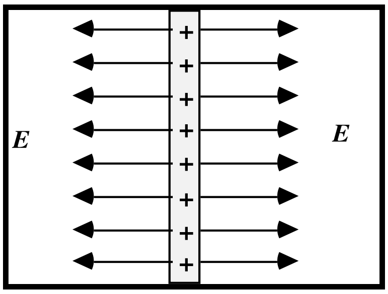


**Figure 2.13 The electric field of a single sheet of positive charge**

By symmetry, the electric field will be directed away from the sheet in both directions as shown. The magnitude of the field is

$$E = \frac{\sigma}{2\varepsilon_0}$$

(i.e. the field lines from *half* the charge go to the right, the other half to the left).

Now introduce a second sheet having a surface charge density of equal magnitude and opposite sign.

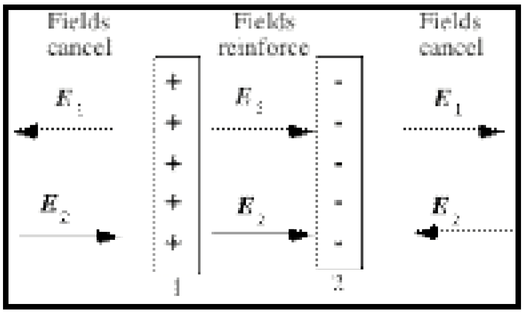


**Figure 2.14 Fields between and beyond two oppositely charged sheets**

*Beyond* the two charge layers the contributions to ***E*** from the layers *cancel.* These are regions of zero field, and therefore equipotential regions.

*Between* the two layers, the contributions add, giving

$$E = \frac{\sigma}{\varepsilon_0}$$

# E3 ELECTRIC CURRENT

## OBJECTIVES□

**Aims**

By the time you have finished with this chapter you should have a good understanding of the nature of electric current and its effects, including the processes of energy transfer and mass transfer. You should also understand the concepts, such as resistance, resistivity and conductivity used to describe the properties of materials, called conductors, which can carry current.

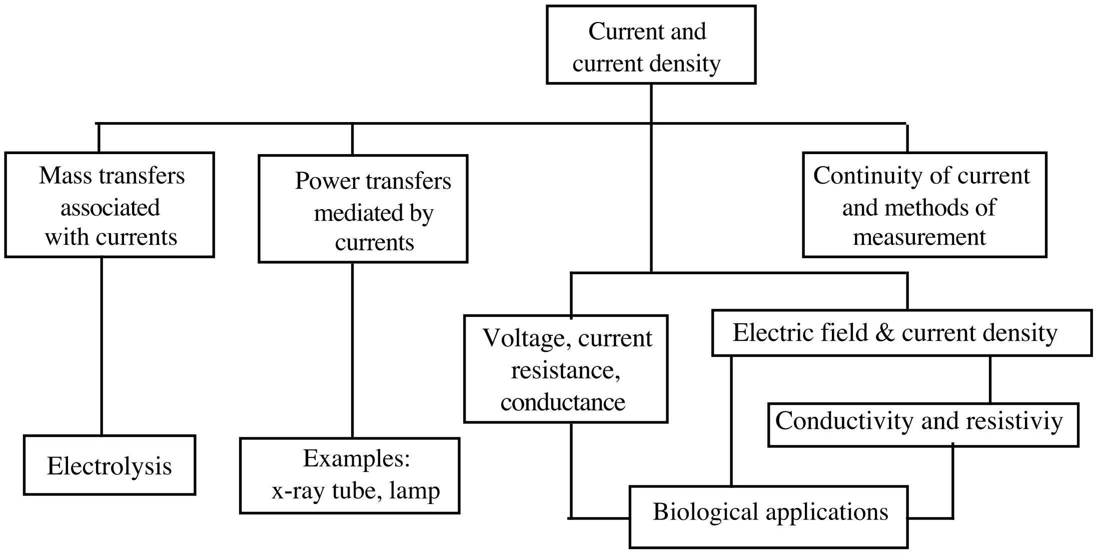
**Minimum learning goals**

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms:

   *conductor, current, ampere, current density, charge carrier, number density, drift velocity, degree of ionisation, mass transfer rate, source, load, electric power, V-I characteristic, resistance, ohm, conductance, siemens, Ohm's law, resistivity, conductivity.*
2. Describe the principal effects of electric currents and ways of measuring currents.
3. State, apply and discuss the equation relating current density to particle densities, charges and drift velocities of charge carriers and explain how the signs of the charge carriers are related to the direction of the current.
4. Give examples of typical magnitudes of current density in engineering and biological applications.
5. Describe and discuss how mass transfers are associated with electric currents. State and apply expressions for mass flow rate in terms of current and the masses and charges of the carriers.
6. State and explain the meaning of Ohm's law for objects and materials.
7. Discuss, explain and apply the relations among current density, electric field, conductivity, resistivity, the dimensions of conducting bodies and resistance.
8. Describe and calculate power transfers in terms of voltage and current.

**CONCEPT DIAGRAM**



# PRE-LECTURE□

## 3-1 THE NATURE OF ELECTRIC CURRENT

An electric current can be broadly defined as charged particles in motion - a flow of charge. The most obvious effect of a current would seem to be the transfer of charge from one place to another. So it is, when a charged body is discharged by touching it with an earthed wire, or when lightning strikes a tree. However, it is important to realise that most currents flow in closed loops or circuits with no net transfer of charge. When there is a steady current the distribution of charge around a circuit remains constant and every part of the circuit remains substantially neutral.

### The effects of electric current

You can't see anything move in an electric current but currents do have a number of extremely important effects, which can be broadly classified as shown in the first column of table 3.1. This list is by no means exhaustive. Some of the topics are discussed in later chapters. In this chapter we are concerned in general terms with the *charge, mass* and *energy* transferred by a current.

| Effect | Applications | Measuring devices |
|---|---|---|
| Heating | Electric room heater | Measure $i^2$ |
| Magnetic | Electromagnet | Clip-on ammeter |
| Electrolysis | Aluminium smelting | Measure □□$dt$ |
| Electrochemical | Kolbe's synthesis of ethane | - |
| Biological | Cardiac defibrillation | - |
| Mechanical (through magnetism) | Moving coil meter<br>Electric motor<br>Loudspeakers | Deflection of a coil |
| Emission of light | Lamp filament<br>Fluorescent light<br>Light emitting diode. | - |
| Voltage drop (potential difference) | Digital ammeter | Measure voltage across a known resistor |

**Table 3.1 Effects of electric current**

### The measurement of electric current

Electric current is measured by means of a device which responds quantitatively to one or other of the effects of the current. Because current is a *flow* of charge, the device usually has to be connected so that the current goes *through* it. The most important effects used in measuring current are listed in the third column of table 3.1.

Each system has advantages and disadvantages. The choice is determined by the exigencies of the situation. For example a clip-on ammeter does not require the interruption of the current, but it is bulky and not very sensitive. By contrast a digital ammeter responds quickly. Moving coil meters have the advantage of simplicity and long-term stability.

### Questions

**Q3.1** To what extent is the flow of water analogous to the flow of charge? What are the hydrodynamic analogues of (a) current, (b) potential difference, (c) resistance? Which of the effects of a current are also shown by flowing water?

**Q3.2** When do you think it might be appropriate to use a current meter based on

a) the heating effect
b) electrolysis?
Give your reasons.

# LECTURE□

## 3-2 ELECTRIC CURRENT AND CHARGE CARRIERS

### Movement of charge

Any movement of charge constitutes an electric current. The charge carriers could be electrons in a vacuum, electrons in a metal, 'holes' in a semiconductor or ions in a solution. The charge may move in free space, through a conductor or on a conveyor belt. An obvious manifestation of charge movement is an electric spark. At each spark some charge is transferred. Together a sequence of sparks makes up an intermittent current in which each spark contributes a current pulse. If the spark gap is narrowed so that the sparks become more frequent, they tend to merge to make a continuous current.

### Definition and unit of current

The value of a current in a wire at any point is defined to be equal to the rate of flow of net positive charge past that point. The direction of the current is defined to be the direction of flow of positive charge.

If the charge carriers are actually negative then the direction of the current is opposite to the flow direction of the particles. Thus a current of one ampere to the right in figure 3.1 could be either positive charge flowing to the right at 1 coulomb per second or negative charge flowing to the left at 1 coulomb per second, or some combination of flows in both directions such 0.5 $C.s^{-1}$ each way.

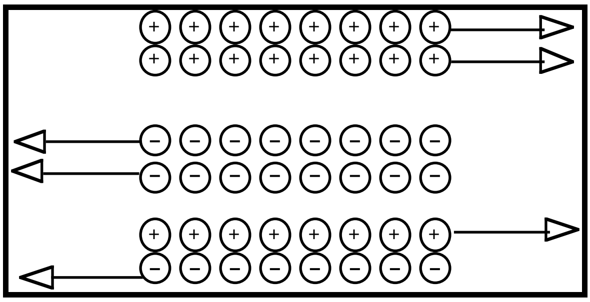


**Figure 3.1 Equivalent currents**

The SI unit of electric current is the ampere (symbol A), equal to one coulomb per second. Because current is easier to measure than charge, the physical standards have been established using current as the base quantity, so the coulomb is defined as an ampere-second (A.s). The ampere is defined in terms of magnetic effects (see chapter E6).

### Kinds of current

The simplest kind of current is that carried by a beam of electrons through a vacuum. For example, in a typical TV tube (figure 3.2), the flow rate of electrons from the electron gun to the screen is about $6 \times 10^{13}$ electrons per second. Since each electron has a charge of $-1.6 \times 10^{-19}$ C, the beam current is about 10 $\mu$A from the screen to the electron gun.

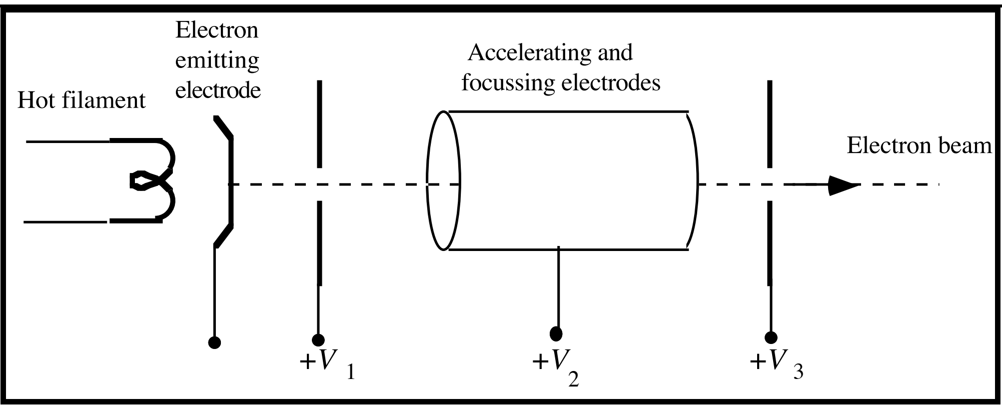


**Figure 3.2 An electron gun**

This one is electrostatically focussed. Many devices, including electron microscopes, use magnetic focussing

Similarly, in wires the positive charges on the ions of the crystal lattice remain fixed, while the current is carried by the drift of the negatively charged electrons. Where confusion might arise you must distinguish between the direction of electron movement and the direction of the conventional current.

Why didn't we avoid this confusion about current direction by calling the electron's charge positive? The answer goes back to Benjamin Franklin who thought he could see which way charge was flowing in sparks. His choice established the definitions of positive and negative in electricity.

In conducting materials only a small fraction of all the electrons is free to move and carry current. The majority are too strongly bound to the respective nuclei. In p-type silicon (and other semiconductors) the number of **mobile electrons** is just less than the number of positive ions in the crystalline lattice of the material. The result is that when there is a current the mobile electrons play a sort of musical chairs, hopping from one vacant point in the lattice to the next. The overall effect can be described as a net flow of positive **holes**. Transistors and other semiconductor devices depend for their operation on simultaneous conduction by electrons and holes.

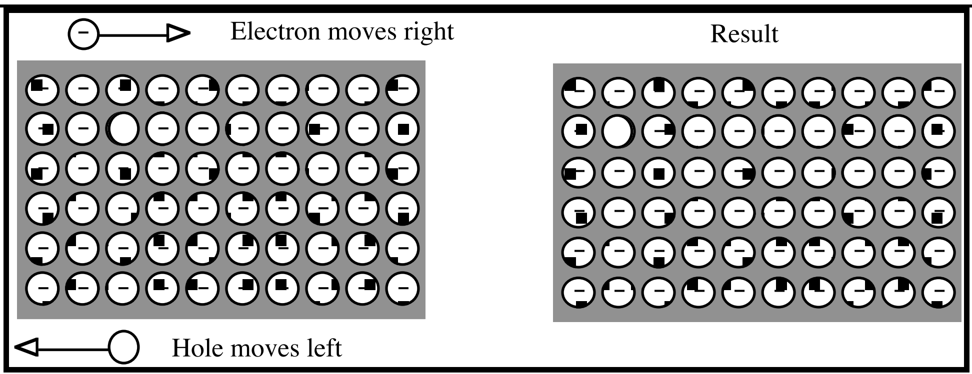


**Figure 3.3 Conduction by positive holes**

An electron, in the 2nd row and 2nd column, jumps into a vacant atomic site, moving from left to right. This is equivalent to saying that the hole moves to the left. The equivalent current is to the left.

## 3-3 CURRENT DENSITY

### Definition and examples

A current in a bulk material is described in terms of the **current density**, that is the current per cross-sectional area of the material. To see what that means think of a small imaginary surface, with area $\Delta A$, inside the material, oriented at right angles to the direction of the current (figure 3.4). Write $\Delta I$ for the current through the surface. The magnitude of the current density, averaged over the surface, is defined as

$$\bar{j} = \frac{\Delta I}{\Delta A} \ . \qquad \text{... (3.1)}$$

If the little surface is allowed to shrink, the average current density approaches a limiting value which is the current density at a point in the material. We need to think of the current density as a vector quantity because we associate direction with currents. So current density is a vector quantity defined at all points in the material - it is a vector field. It varies in magnitude and direction throughout a body and it can also be mapped in the same way as the electric field by drawing lines of current density which show the direction of the current at each point in the material. In fact, unless the material is a superconductor, current cannot exist without an electric field and the directions of the electric field and the current density must be the same at every point.

Rain often carries current. In this instance the charge carriers are raindrops. In a typical storm, the current carried to each square meter of ground may be 0.1 $\mu$A. This is expressed by saying that the current density in rain is 0.1 $\mu$A.m$^{-2}$. By contrast, the current densities through the membranes of living cells are in the region of 1 A.m$^{-2}$. Electric power conductors carry current densities of the order of 0.1 GA.m$^{-2}$.

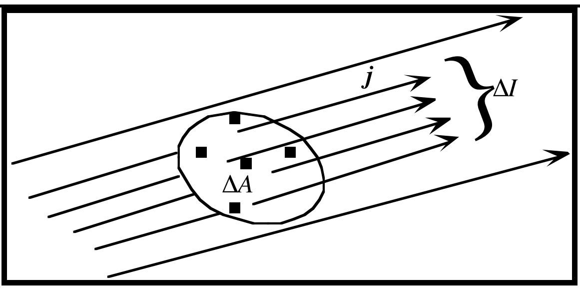


Figure 3.4 Current and current density

### Theory: current density and charge flow

The current density in a material depends on the density of the charge carriers and their average velocity at each place. The particles which carry current in a material - electrons or ions - are continually moving at high speeds, typically some hundreds of metres per second, but if there is no current they don't actually go anywhere on average. They are continually colliding and bouncing off each other and other particles, so they are just as likely to be moving in one direction as in any other. Their average velocity, averaged over all the charge carriers and any reasonable time interval, is zero. When a current exists, the charge carriers still rush around madly, but between collisions they acquire a little extra velocity in a definite direction - parallel to the electric field if they are positive and antiparallel if they are negative. This extra component of velocity is called drift velocity. Surprisingly perhaps, drift velocities are quite small, typically some fraction of a millimetre per second. What we think of as large currents are large because they involve a huge number of charge carriers.

We might expect that current density $j$ should increase if we could increase any of the three quantities, the number density of charge carriers (number per volume, $n$), the charge on each carrier ($q$) and the magnitude of the drift velocity ($\bar{v}$). In fact it is given by the product of all three which is the only combination, apart from a possible numerical factor, that gives the correct units:

$$j = nq\bar{v} \quad . \qquad \text{... (3.2)}$$

The same result can be worked out more formally by finding the rate at which the charge carriers go through a small imaginary surface of area $A$ (represented by the white area in figure 3.5). In a short time interval d$t$ all those carriers which are within a distance $\bar{v}\,\mathrm{d}t$ of the surface will get through. The number of particles is the number within the volume $A\bar{v}\,\mathrm{d}t$, that is a total of $nA\bar{v}\,\mathrm{d}t$ particles. They carry a total charge of $qnA\bar{v}\,\mathrm{d}t$. To find the current divide the charge by the time interval: $I = nq\bar{v}A$. Finally, divide by the area to get equation 3.2.

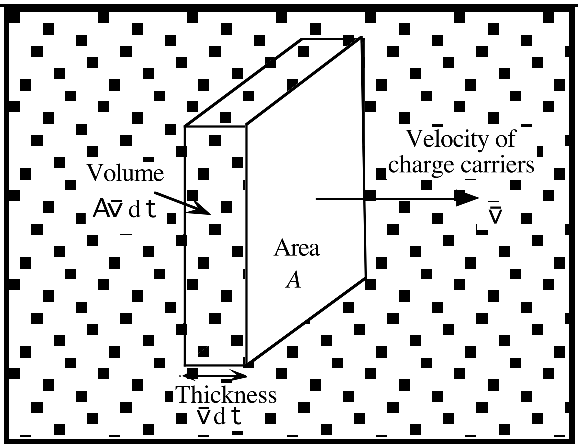


Figure 3.5 Current density and drift velocity

In electrolytes, ionised gases, living cells and so on there will in general be more than one type of charge carrier, and each will contribute a term to the current density. For two carrier types,

$$\boldsymbol{j} = n_1 q_1 \boldsymbol{v_1} + n_2 q_2 \boldsymbol{v_2} \qquad \text{... (3.2a)}$$

(Note that $\boldsymbol{j}$, $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are vector quantities. If there is only one kind of carrier then the current density vector has the same direction as the drift velocity of positive charge carriers.)

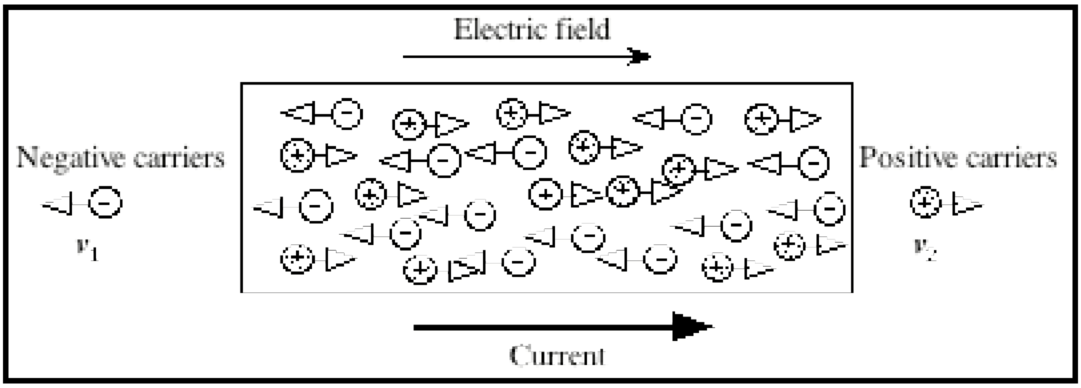


**Figure 3.6 A current of moving positive and negative carriers**
In this diagram, the positive carriers drift to the right and the negative carriers drift to the left. Both sets of carriers contribute to the current to the right

**Demonstration: Practical application of current as an analogue of water flow**

The flow of charge is analogous to the flow of fluids. In the video lecture, Professor Collis-George demonstrates how this analogy is put to practical use. Current in electrically conducting paper is used to predict patterns of water flow in proposed soil drainage systems.

The conducting paper is cut to the shape of the region through which the water will flow, and metal electrodes are placed on this paper to represent the ditches through which the water flows, into and away from this region. The positions of several points on a specific equipotential are located with a probe attached to a voltmeter, and this equipotential is drawn. Further equipotentials between the electrodes are drawn in this manner, and give the water isobars within the irrigated soil. The electric current is in the direction of the electric field. Thus, by drawing lines *perpendicular* to these equipotentials (isobars) 'streamlines' are obtained which show the pattern of the current between the electrodes.

By using this model we can obtain the water pressure within the soil, the pattern of water flow, and even the total volume of water that flows through the soil. Thus the effectiveness of an irrigation system can be checked without the effort of digging ditches or of measuring the water pressure profile in the irrigated region.

### Ions

Since we usually consider the charge carriers to be either electrons or ions we can express their charges as multiples of the electronic charge $e$; $q = ze$. The integer $z$ is called the degree of ionisation. So for example an electron has $q = -e$, and a doubly ionised positive ion such as copper ($Cu^{++}$) has $q = 2e$.

## 3-4 ELECTRIC CURRENT AND FLOW OF MASS

Not only is electric current analogous to fluid flow: all charge carriers have mass, so a mass flow is necessarily involved in any electric current. Of course, in some circumstances the *net* mass flow may be zero (how?). Electric current is sometimes used to produce a mass flow.

**Demonstration**

An example explained in the video lecture is the vacuum coating of glass with copper by the process of sputtering. This is one stage in the production of one type of solar energy collector. It works as follows.

A heavy current of argon ions is established in a vacuum tube. Since the ions have mass, the current carries energy and momentum. The ions strike a copper electrode with sufficient kinetic energy to dislodge a stream of copper atoms which fly through the vacuum and coat the collector tube with copper.

## How much mass is carried by a current?

Since current is just the charge flow rate, it is easy to work out the rate at which mass is transferred by a particular kind of charge carrier:

$$\frac{\text{mass flow rate}}{\text{charge flow rate}} = \frac{\text{mass of particle}}{\text{charge of particle}}$$

Substituting $I$ for the current (charge flow rate), $m$ for the mass of the carrier and $z$ for its degree of ionisation, we can express the mass flow rate ($\mu$) as

$$\mu = \frac{Im}{ze} \; . \qquad \text{... (3.3)}$$

For a current of 1 A carried by electrons the mass flow rate is about 6 ng.s$^{-1}$ (in the opposite direction to the current)! In a lightning discharge, the *mass* of the electrons transferred is of the order of picograms. Currents carried by electrons transfer relatively small masses, a consequence of the small mass of the electron. Much larger masses are transferred by ions in the process of electrolysis, which will be considered in chapter E5.

### Description in terms of molar quantities.

While physicists talk about the charge and mass of individual ions, chemists use the charge and mass of a *mole* of ions. The mole is the amount of substance of a system that contains as many elementary entities ($6.02 \times 10^{23}$) as there are atoms in exactly 12 g of pure carbon-12. The elementary entities may be atoms, ions, electrons, molecules, etc.

The molar mass of a substance is the mass of a sample divided by the amount of the substance and its SI unit is the kilogram per mole (kg.mol$^{-1}$). For example a mole of water has a mass of 18 g and a mole of sugar has a mass of 342 g. The number of atoms per mole is called the Avogadro constant.

The **Faraday constant**, denoted by the symbol $F$, expresses the ratio of charge to amount of substance for singly ionised ions and is defined as

$$F = 9.6 \times 10^{4} \text{ C.mol}^{-1}$$

$$= 6.02 \times 10^{23} \text{ electronic charges per mole.}$$

Hence, in terms of molar quantities, the mass flow rate associated with a current is

$$\text{mass flow rate, } \mu = \frac{IM}{zF}$$

where $M$ is the molar mass. This expression is just the same as equation 3.3 with both numerator and denominator multiplied by the Avogadro constant.

***Demonstration***

In the video lecture, the purification of copper by electrolysis is illustrated. Since the copper ion is Cu$^{++}$ (molar mass $63 \times 10^{-3}$ kg.mol$^{-1}$), the mass flow rate produced by a current of 1 kA is

$$\text{mass flow rate} = \frac{(1 \times 10^{3}\text{ A}) \times (63 \times 10^{-3}\text{ kg.mol}^{-1})}{9.6 \times 10^{4}\text{ C.mol}^{-1} \times 2} = 3 \times 10^{-4} \text{ kg.A.C}^{-1}$$

$$= 0.3 \text{ g.s}^{-1}.$$

Huge currents are used in commercial electrolytic smelting and purification of metals.

## 3-5 CURRENT AND ENERGY FLOW

### Current and energy transfer

When there is an electric current in a circuit, energy is generally being transferred from a **source** to a **load**: energy is transferred from a battery to a lamp or from a nerve cell to measuring electrodes. In such circuits, the current is associated with the energy transfer.

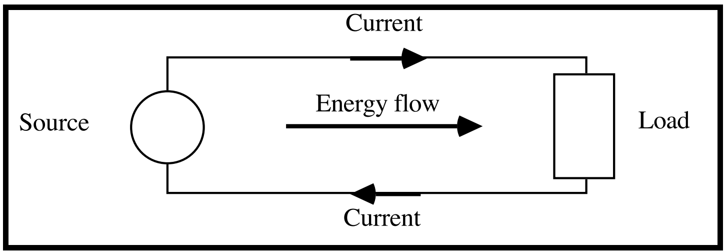


**Figure 3.7 The transfer of energy in a simple circuit**

The charge flows continuously round the circuit, from source to load and back again. The energy flows from source to load. From this it is clear that the current is not the energy, but is simply associated with transfer of energy. (In rather the same way a bicycle chain is the means by which muscle energy is transferred from the crank to the wheel.)

The intermediate step in conveying energy from the source to the load is *electric potential energy*. The electrons gain potential energy in the battery, and lose this potential energy in the load.

If the source were connected to an evacuated tube such as a cathode ray tube this potential energy would be converted to the kinetic energy of the electrons.

When a source is connected to a lamp (or any other resistive load) electric potential energy is also given to the charge carriers (electrons) by the source. However as they go round the circuit they almost immediately start to lose energy in collisions with the ions which make up the crystalline structure of the conducting metal. This lost energy causes the material to heat up, even to the extent that it emits light.

**Demonstration: Rolling marbles model of current**

In the marble model of a simple circuit demonstrated in the video lecture, the muscle energy expended in raising the gravitational potential energy of the marbles corresponds to the chemical energy expended by a battery in raising the electrical potential energy of electrons.

Initially, as they roll down the ramp, the marbles gain kinetic energy that is equivalent to the gravitational potential energy they lose. This is almost immediately lost in collisions with the pegs on the ramp, appearing as sound and internal energy. A marble rolling down the ramp gains, on the average, as much kinetic energy between collisions as it loses per collision.

Notice that there is not much potential drop across the supply and return ramps. These ramps represent the low resistance wires connecting the source to the load.

### Electric power (= rate of energy transfer)

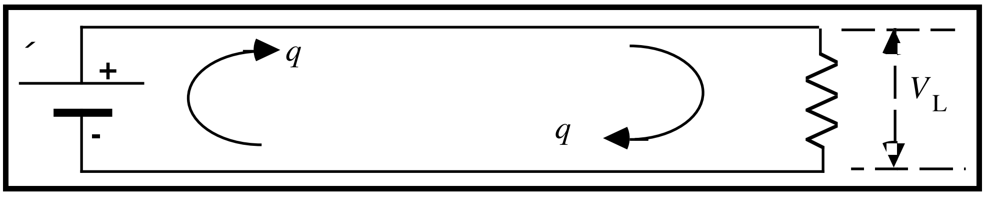


**Figure 3.8 Power transfer in a circuit**

Consider a charge $q$ which flows once round the circuit shown in figure 3.8. The EMF of the source is ′. When a charge $q$ traverses the circuit, it gains energy $+'q$ as it goes through the battery (source) and loses energy $+V_L q$ (as light and heat) when it passes through the load. Usually these energies are almost equal as there is little energy lost in the connecting wires. (Energy is conserved: the energy gained in the source is equal to the energy lost in the load and connecting wires.)

If there is a continuous current $I$, the *power* delivered (i.e. the *rate* at which energy is delivered) by the source is

$$P_s = \frac{d}{dt}(\mathcal{E}q) = \mathcal{E}\frac{dq}{dt} = \mathcal{E}I .$$

Similarly the power *consumed* by the load is

$$P_L = V_L I .$$

In general, for any component the power transferred is the product of the voltage across it with the current through it:

$$P = VI . \qquad \text{... (3.4)}$$

## 3-6 RESISTANCE AND RESISTIVITY

### Measurement of *V-I* characteristics

The relationship between the current through an object and the potential difference across it can be explored experimentally by varying the applied voltage and plotting a graph of $V$ against $I$.

**Demonstration**

In the video lecture, an X-Y chart plotter is used to plot such graphs quickly.

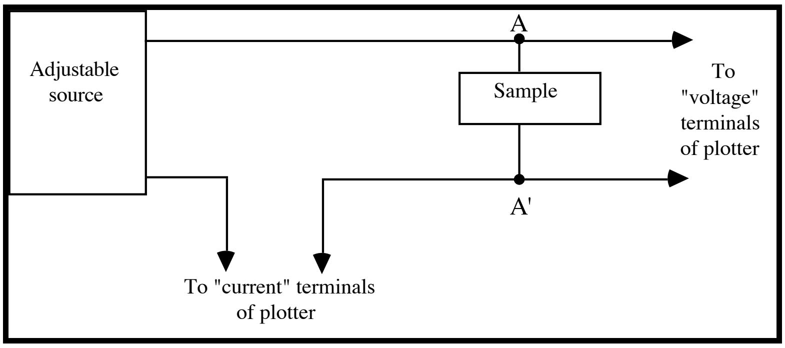


**Figure 3.9 Apparatus for plotting *V-I* graphs**

The variable voltage $V$ across the sample deflects the pen in the Y direction (in this particular experiment $-5\text{ V} < V < 5\text{ V}$) and the corresponding current $I$ deflects the pen in the X direction ($-100\text{ mA} < I < +100\text{ mA}$). Thus at any time the pen position records both $V$ and $I$. By adjusting the variable voltage supply, a graph of $V$ against $I$ is automatically plotted for the device or sample connected at AA'. Graphs were thus plotted for a coil of copper wire, a small lamp and a semiconductor diode with the results shown in figure 3.10.

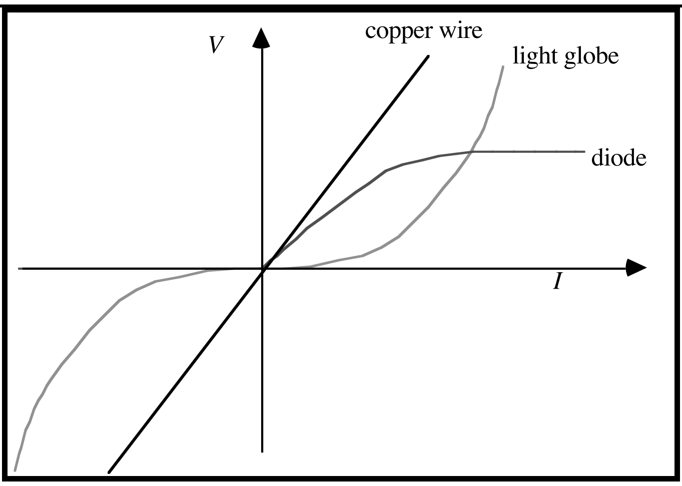


**Figure 3.10 V-I graphs for a wire, a lamp and a diode**

For the copper wire, the *V-I* characteristic was a straight line of uniform slope. The slope of the line is equal to the **resistance**, *R,* of the sample

$$R = \frac{V}{I} = \text{constant} \qquad \text{... (3.5)}$$

An object which is a poor conductor has a high resistance. An alternative way of describing the conducting ability of an object is to specify is conductance $G$ which is defined simply as the reciprocal of resistance. One advantage of the idea of conductance is that it gives sense to the rule for combining electrical objects connected in parallel; parallel paths conduct better so you just add conductances.

The SI unit of resistance is the volt per ampere, which has the special name, ohm, (symbol Ω). The unit of conductance is the siemens (S), which is equivalent to a reciprocal ohm.

### Ohm's law

**Ohm's law** states that for many substances, over a wide range of conditions, the resistance of an object is constant, as it was for the copper wire sample. The copper wire sample is said to obey Ohm's law. However, the lamp and the diode do not obey Ohm's law.

The lamp had a low resistance (small *V/I*) at first (i.e. small applied voltage). However as the current increased, the voltage increased more rapidly. As the lamp heated up, its resistance increased, as evidenced by the increasing slope of the graph. (Note: Ohm's law generally applies to metals at *constant* temperature.)

The diode is another example of a **non-ohmic** device. The resistance change is not now due to a temperature change; in fact the diode is an almost perfect 'one way' device. For positive voltages above about 0.5 V, the resistance (*V/I*) is very low, and the diode is an excellent conductor. For voltages of the opposite polarity, the resistance is very high: the diode hardly conducts at all. Notice that we still talk about the resistance of non-ohmic devices. The difference is that the resistance is not constant, but varies with current. For such devices, the resistance read by an ohmmeter will depend on the current used in making the measurement.

### Application to bulk materials: resistivity and conductivity

So far we have discussed the resistance of samples or devices. For bulk materials, it is more pertinent to look at what is happening inside the material, in terms of current density and what determines its value. It has been noted already that at each point in the material the current depends on the electric field and is in the same direction as the field. Except for superconductors, no field, no current. The relation between field and current density can be *defined* as:

$$\mathbf{j} = \square \mathbf{E} \,. \qquad \text{... (3.6)}$$

Here □ represents a property of the material called its **conductivity**. (Note: some books use the symbol □ for conductivity, but we have already used that symbol for surface charge density.) Equation 3.6 does not imply that current density is proportional to field, the conductivity could, and often does, depend on the field. However it is a constant for some for some materials when they are held at constant temperature. They are the same materials which obey Ohm's law, so saying that □ is constant is stating that Ohm's law applies. It is often convenient to use the property **resistivity**, □ which is just the reciprocal of conductivity:

$$\square = \frac{1}{\square} = \frac{E}{j} \qquad \text{... (3.7).}$$

Remember that conductivity and resistivity are properties of a material rather than an object with a particular shape. The SI units are siemens per metre ($\text{S.m}^{-1}$) for conductivity and ohm metre (Ω.m) for resistivity. Table 3.2 gives the resistivities and conductivities of a few common materials.

| **Substance** | **□ / (Ω.m)** | **□/ ($\mathbf{S.m^{-1}}$)** |
|---|---|---|

| | | |
|---|---|---|
| Copper | $1.6 \times 10^{-8}$ | $6 \times 10^{7}$ |
| Living tissue | 0.2 | 5 |
| Pyrex glass | $\sim 10^{14}$ | $\sim 10^{-14}$ |
| Distilled water | $10^{2}$ to $10^{5}$ | $10^{-5}$ to $10^{-2}$ |
| Fused quartz | $> 10^{14}$ | $< 10^{-14}$ |

**Table 3.2 Some typical conductivities and resistivities**

### Resistivity, resistance and sample shape

The resistance of an object depends on the materials used to make it, its size, its shape and the places where connect on to it. The last point is important. A given object can many different values of resistance, because the current you get depends not only on the voltage applied but also on the places where you let the current in and out. Calculating the resistance of a particular arrangement could be a daunting task, but there is one simple case that is easy to do, and is quite useful. It requires a specimen which has a uniform cross section and electrical connections to the two ends which admit the current uniformly over the end faces. (A metal coating whose conductivity is much higher than that of the object, spread over each end would do the trick.)

So consider a sample of uniform cross-sectional area $A$ and length $l$. A current $I$ passes through the sample as shown in figure 3.11, and the corresponding potential difference between its ends is $V$.

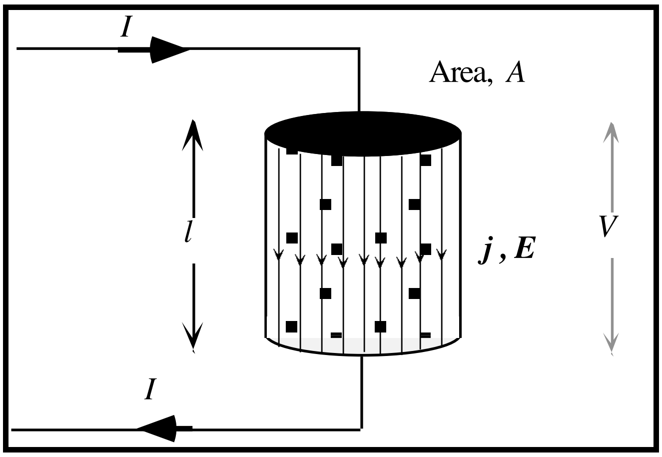


Figure 3.11 Measurement of resistivity

Provided the current density is uniform, its magnitude is given by

$$j = \frac{I}{A}$$

and the electric field (parallel to $\boldsymbol{j}$) has a magnitude

$$E = \frac{V}{l} \, .$$

Now in the material, $E = \rho j \, ;$

hence, substituting for $E$ and $j$ we obtain:

$$\frac{V}{l} = \rho \frac{I}{A} \, .$$

Now use the definition of resistance $$R = \frac{V}{I}$$

to find that $$R = \rho\frac{l}{A}\ . \qquad \text{... (3.8)}$$

This resistance is proportional to the length and inversely proportional to the cross-sectional area.

**Demonstration**

Conductivity measurements of an aqueous extract of soil prepared under standard conditions are used to determine the salt content of the soil, and hence the suitability for various crops. The technique is demonstrated in the video lecture by Prof. Collis-George. The resistance of a cubical sample is measured. The meter is calibrated directly in conductivity for that particular size of cubical sample. (If the cube is of side $a$ what is the conversion factor?)

## 3-7 EXAMPLE - ELECTRIC CURRENT IN AN X-RAY MACHINE

An x-ray machine represents the first contribution of modern physics to medicine. Its mode of operation involves many of the principles discussed in this chapter.

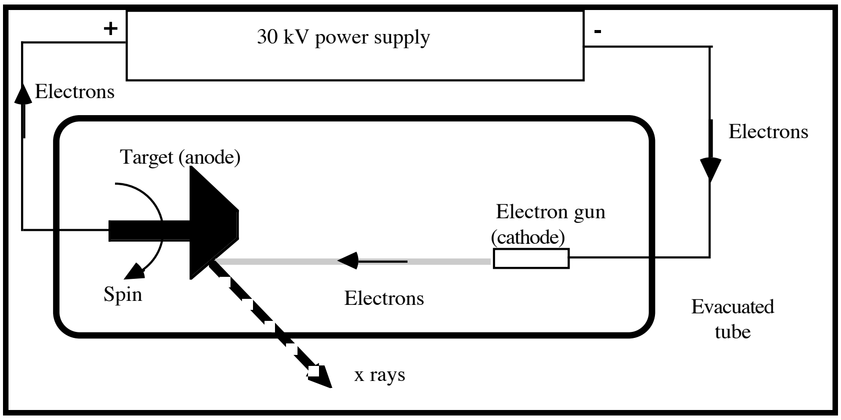


**Figure 3.12 The basic components of an x-ray machine.**

During the exposure, there is a uniform current of about 0.1 A in the circuit. In the wires this is carried by electrons, moving at a uniform average speed of about 10 $\mu$m.s$^{-1}$. However in the x-ray tube the electrons are accelerated from low speeds when they leave the electron gun to a speed of about $3 \times 10^7$ m.s$^{-1}$ when they strike the target. Since the current has the same value at all points along the beam the number of electrons per length of beam thins out correspondingly as the electrons move towards the target. At all points in the circuit,

$$I = \text{const} = n_l ev$$

where $n_l$ is the number of electrons *per length* of the wire, and of the beam within the x-ray tube.

The purpose of the power supply (an inappropriately named device for a supplier of energy) is to drive the current round the circuit. As they pass through the power supply, the electrons gain *electric potential energy* of 30 kilojoules for each coulomb of charge. A convenient (non-SI) unit for specifying the energy of one electron is the electronvolt (symbol eV), defined as the change in potential energy of one electron when it moves through a potential difference of one volt. So each electron moving through power supply (EMF 30 kV) gains PE of 30 keV.

The wires are good conductors, so there is little change in the energy of the electrons in the wires. However in the x-ray tube, the electrons are free to accelerate. In passing from the electron gun to the target they lose 30 keV of potential energy and gain an equal amount of kinetic energy. On striking the anode, the electrons are abruptly stopped. Most of their energy is eventually converted into internal energy of the target. A small fraction is converted into useful x-ray energy.

So much internal energy is transferred to the target that there is a chance of rapid damage to its surface. To avoid hot-spots the anode is spun rapidly during the exposure. That is why x-ray machines make a characteristic whirring sound.

# POST-LECTURE□

## 3-8 CORRECT USE OF ELECTRICAL TERMINOLOGY

Professor Bragg Snr of the University of Adelaide is reputed to have once asked his class, ‘who can tell me just what electricity is?’ and received the reply, ‘I did once know, sir, but I've forgotten’. ‘What a tragedy for Mankind’, said Bragg, ‘the only person who ever knew what electricity was has forgotten!’

Electricity *is* mysterious. You can't see or weigh it. It needs curious unnatural machinery to generate it. It does all sorts of extraordinary things. This perceptual mystery of electricity is no excuse for failure to understand and use correctly the few technical terms used in descriptions of electricity. Expressions such as ‘the resistance caused a high amperage across the battery’ and ‘it must have had a lot of volts through it’ are not only quite meaningless; they reveal a profound misunderstanding of basic physical ideas involved. Unfortunately such misuse of electrical terminology is all too common.

Here are a few terms and ideas which you must thoroughly understand.

- Electric current is the flow of charge. It is measured by inserting an ammeter in series with the circuit. There is no such word as *amperage*. The term is *current* and its unit is ampere.
- Potential difference is defined in chapter E2. It is analogous to pressure difference and is measured by connecting a voltmeter between the two points of interest. Usage permits us to use the technician's term 'voltage' for potential difference. The unit of potential difference is volt.
- The SI unit of power is the watt (but power is not called *wattage*). Electric power = potential difference □ current.
- The SI unit of energy is the joule. A joule is a watt second, or alternatively a watt is a joule per second. For reasons of convenience, electrical energy is sometimes measured in kilowatt hours: 1 kW.h = 3.6 MJ.
- The resistance of a component is the ratio of the potential difference across the terminals to the current in the component. The SI unit of resistance is volt per ampere which has the special name of ohm. There is no such word as 'ohmage'.

## 3-9 QUESTIONS

**Q3.3** If electrical energy costs 18 c for each kW.h, what would it cost to raise a house of mass 500 tonnes a height of 1 m above the ground, using an electric motor of 50% efficiency?

**Q3.4** a) Sketch an x-ray tube and label the electron gun and anode. Indicate the polarity of the accelerating voltage, the direction of electron flow and the direction of the current.

b) Consider an x-ray exposure of 0.10 s with a beam current of 0.20 A and an accelerating voltage of 30 kV. What charge is transferred during the exposure? How many electrons strike the anode?

c) How much mass is transferred during the exposure?

d) At what rate does the power supply deliver energy during the exposure?

e) How much energy is released in the exposure?

f) Calculate the speed and energy of each electron just before it strikes the anode.

g) Why is the anode of an x-ray tube spun during the exposure? Roughly how fast should the anode spin in terms of the exposure time?

**Q3.5** In an experiment to measure the membrane potential of a living plant cell, micro-electrodes are placed inside and outside the cell. Making the measurement produces a current of 1.5 nA through the membrane from the exterior to the interior of the cell. The total area of the cell membrane is 1.0 mm$^2$. Calculate the current density in the membrane produced by the measurement.

If the current is carried through the membrane equally by chloride ions Cl$^-$ and sodium ions Na$^+$, calculate the flow of mass of each ion species during the measurement, and the number of ions transferred in a measurement lasting 30 s.

$$\text{Mass of a Cl}^- \text{ ion} = 5.9 \times 10^{-26} \text{ kg.}$$

$$\text{Mass of a Na}^+ \text{ ion} = 3.8 \times 10^{-26} \text{ kg.}$$

**Q3.6** In a storm, falling raindrops carry a current density of $1.0 \times 10^{-6}$ A.m$^{-2}$ towards the ground. If 100 drops strike each square meter of ground per second, calculate the magnitude and sign of the average charge on each raindrop.

**Q3.7** In a copper (Cu$^{++}$) electrolysis tank, what mass of copper is deposited by a current of 10 kA in 10 minutes?

$$\text{Mass of Cu}^{++} \text{ ion} = 1.1 \times 10^{-25} \text{ kg.}$$

**Q3.8** A laboratory maintains a group of animal houses which are heated electrically by radiators with a combined capacity of 100 kW. The radiators are controlled by thermostats. On average, the radiators are on about 50% of the time in the four coldest months, and about 10% of the time during the rest of the year. If electrical energy costs 18 cents per kW.h, what is the annual cost of heating the animal houses?

**Q3.9** a) A research assistant has been making measurements of voltage and current on a sample of living material, and presents you with the following data:

| Voltage/V | Current/$\mu$A |
|---|---|
| 3.0 | 150 |
| 2.0 | 110 |
| 1.0 | 45 |

What can you deduce about the resistance of the specimen from these data? What, in your view, is the most serious deficiency in your assistant's work?

b) A more experienced assistant finds by experiment that the relationship between current and voltage for a different specimen can be expressed as

$$V = aI + bI^2$$

where $a = 70$ V.A$^{-1}$ and $b = 2.0 \times 10^3$ V.A$^{-2}$.

In addition to the resistance, physicists often use a quantity called dynamic resistance to describe the properties of an object like this one. The dynamic resistance is defined as the rate of change of voltage with respect to current which can be found from the slope of a *V-I* graph, rather than the usual ratio *V*/*I*.. To avoid confusion the usual resistance can then be referred to as the static resistance. What are the static resistance and the dynamic resistance for this object at a current of 10 mA?

**Q3.10** An aqueous extract of a particular soil is placed in a glass tube of internal diameter 5.0 mm and length 0.64 m. The resistance between the ends of this column of extract is 2.15 kΩ. Calculate the resistivity and the conductivity of the solution.

**Q3.11** Given that current density in a material must depend on the number density of charge carriers, the charge on each one and their drift velocity, show that apart from an unknown numerical factor, equation 3.2 is the only one that balances the units correctly.

# E4 ION DIFFUSION

## OBJECTIVES☐

### Aims

By studying this chapter you should aim to understand the physical processes which lead to the existence of membrane potentials for both equilibrium and non-equilibrium situations. You should be able to explain these processes in your own words and apply the ideas to examples and simple quantitative problems.

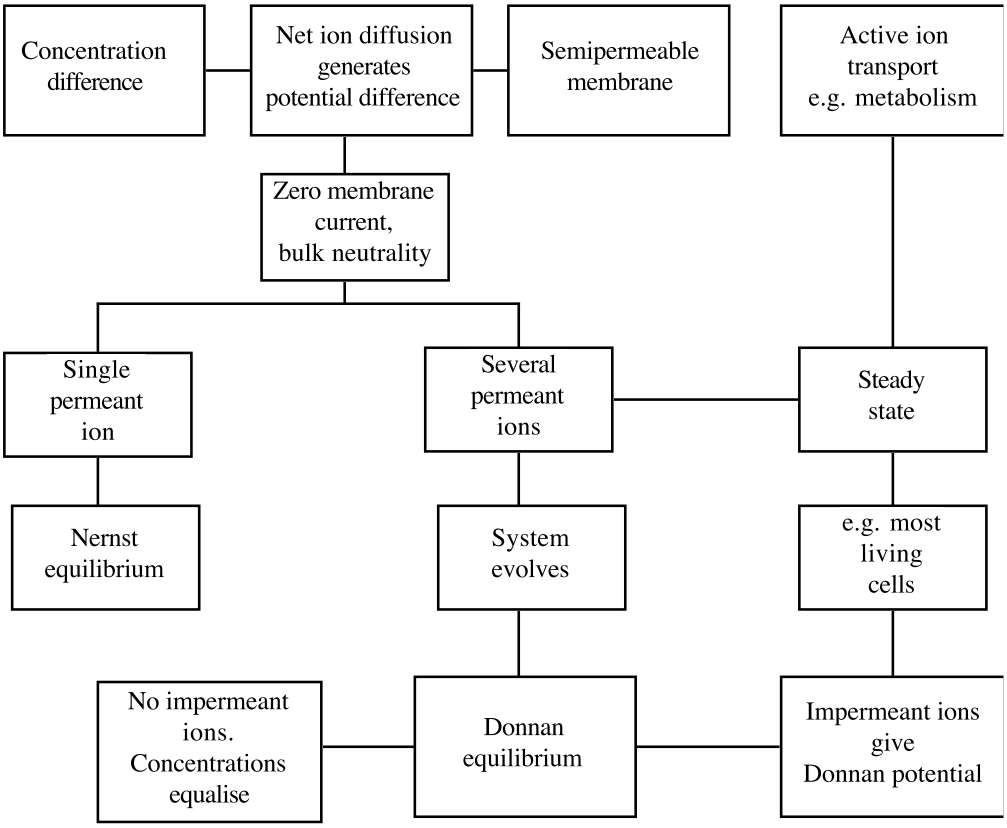
### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms:

   *concentration, concentration ratio, number density, pH, ion diffusion, semipermeable membrane, permeant ion, diffusion potential, Nernst equilibrium, Nernst potential, Nernst equation, ion pump, sodium-potassium pump, junction potential, Donnan equilibrium, Donnan potential.*
2. Describe and explain the processes of ion diffusion and Nernst equilibrium
3. Describe and discuss the roles of ion diffusion, ion pumps and stable diffusion potentials in living organisms.
4. State and use the Nernst equation.
5. Describe and explain the operating principles of a pH meter.
6. Distinguish between equilibrium and non-equilibrium ion diffusion potentials and explain the conditions which give rise to a Donnan equilibrium.

### CONCEPT DIAGRAM



# PRE-LECTURE

## 4-1 CONCENTRATION AND pH

### Amount of substance

Mass is a common way of specifying how much material you have, but for some purposes it is much more pertinent to think in term of how many particles you have. Since the atoms or molecules of different substances have different masses, equal masses of different substances contain different numbers of particles. It is useful to define a new quantity, **amount** of substance, such that equal amounts of two different substances have the same number of elementary entities - atoms, molecules or whatever you consider the basic unit of each substance to be. The concept of amount is useful whenever you are studying something that depends on the number of particles present rather than on their masses. The SI unit of amount is called the mole (symbol mol) and is defined to be the amount of substance containing the same number of elementary entities as there are atoms in exactly 12 g of the pure isotope carbon-12. It turns out that there are $6.022 \times 10^{23}$ particles per mole. The conversion factor is known as the Avogadro constant $N_0 = 6.022 \times 10^{23}\ \text{mol}^{-1}$. (The term Avogadro's number, meaning the pure number $6.022 \times 10^{23}$, is regarded as obsolete, since its value depends on a particular system of units, whereas the Avogadro constant is a constant of nature whose value is independent of human invention.)

You can imagine an amount of any discrete entity you like; a mole of water molecules, a mole of electrons, a micromole of sand grains or even a nanomole of bacteria. However the concept of amount is most useful when we are dealing different kinds of particles whose numbers must conform to simple integer ratios, such as atoms in pure chemical compounds. To take a simple example used in this chapter, any amount the chemical compound potassium chloride (KCl) contains exactly the same number of potassium (K) atoms and chlorine (Cl) atoms. So KCl is formed from equal amounts of potassium and chlorine (not equal masses).

### Concentration

There are several possible ways of defining or specifying the concentration of a solution. We could for example, specify the mass of solute per mass of solvent, or amount of solute per volume of solution. For the understanding of electrical conduction and other electrical processes at the atomic scale we use the following concepts of number density and concentration.

**Number density** (*n*) means number of particles per volume:

$$n = \frac{\text{number of particles in solution}}{\text{volume of solution}} .$$

This idea has already been used to describe conduction by charge carriers in chapter E3.

Amount of substance per volume, called **concentration**, is an alternative concept for dealing with phenomena that depend on number densities of particles.

$$C = \frac{\text{amount of ion species}}{\text{volume of solution}}$$

Concentrations specified in this way are often referred to as molar concentrations. Note that the SI unit of concentration is the mole per cubic metre. A more practical unit (but non-SI) is the mole per litre ($\text{mol.L}^{-1}$). A solution with a concentration of 1 mole per litre is often referred to as a molar solution; a concentration of $1.00 \times 10^{-3}$ mole per litre would be called millimolar (mM).

Number density and concentration are connected by the Avogadro constant :

$$n = N_0 C .$$

### The pH of a solution

The **pH** of an aqueous solution is a pure number which expresses its acidity (or alkalinity) in a convenient form widely used in the life sciences. (The pH of a solution is literally its ‘hydrogen power’.) It is a measure of the hydrogen ion concentration expressed on a logarithmic scale.

The pH of a solution is defined to be $-\log_{10}\left[C_{H^+}/(\mathrm{mol.L^{-1}})\right]$. Note that the value of pH is linked to the chosen unit of concentration, mole per litre.

**Example** Pure water is partly dissociated into $H^+$ and $OH^-$ ions according to the chemical equation:

$H_2O \rightleftharpoons H^+ + OH^-$. (The ions actually attach themselves to water molecules, but in terms of counting charged particles that makes no difference.)

If the concentration of $H^+$ is increased by adding an acid to pure water the equilibrium is disturbed; the additional $H^+$ ions cause some of the $OH^-$ ions to recombine, thus decreasing the concentration of $OH^-$ ions. Similarly the addition of an alkali causes a reduction in the concentration of $H^+$ ions.

For an aqueous solution in equilibrium, the rate of dissociation of $H_2O$ must be equal to the rate of recombination of $H^+$ and $OH^-$, which is proportional to the product of the concentrations of $H^+$ and $OH^-$. By equating these two rates it is easily shown that

$$C_{H^+} \times C_{OH^-} = K \quad \text{( a constant)}$$

Thus by measuring the concentration of hydrogen ions we can determine the acidity (or alkalinity) of a solution.

For pure water $C_{H^+} = C_{OH^-} = 1.0 \times 10^{-7}\ \mathrm{mol.L^{-1}}$,

so the constant in the equation is $K = (1.0 \times 10^{-7}\ \mathrm{mol.L^{-1}}) \times (1.0 \times 10^{-7}\ \mathrm{mol.L^{-1}})$

$= 1.0 \times 10^{-14}\ \mathrm{mol^2.L^{-2}}$.

Thus in terms of pH we have $C_{H^+} = 1.0 \times 10^{-pH}\ \mathrm{mol.L^{-1}}$

and $C_{OH^-} = 1.0 \times 10^{(pH-14)}\ \mathrm{mol.L^{-1}}$.

#### *Values of pH*

- pH < 7 corresponds to an acid solution, i.e. one in which $C_{H^+} > C_{OH^-}$.
- pH = 7 corresponds to a neutral solution, i.e. one in which $C_{H^+} = C_{OH^-}$.
- pH > 7 corresponds to an alkaline solution, i.e. one in which $C_{H^+} < C_{OH^-}$.

In practice pH occurs in the range 0 < pH < 14; a much narrower range (around 7) is encountered in living systems.

**LECTURE**

## 4-2 ELECTRICITY IN THE BIOLOGICAL WORLD

In all living organisms, from the simplest bacterial cells to humans, electric fields, potentials and electric currents play a fundamental role. Electricity plays an analogous role in biological systems to electric signalling and electric power in modern civilisation. Whereas in modern civilisation electricity mostly comes from generators that produce electrical energy by electromagnetic induction, in living things the electricity originates at a molecular level, by the diffusion of ions through a membrane, or by proteins which expend chemical energy, and in the process separate electric charges.

In rapidly growing organisms such as the pink bread mould *nirospira* a quarter or more of the cells' energy budget is expended at the surface membrane. This energy goes into driving electric currents through the membrane, which are utilised in feeding the cell through electrically charged nutrients.

All cells that respire - i.e. nearly all cells in the biosphere - do so by means of an organelle within the cell called the *mitochondrion*. The respiration process involves the combustion of a chemical fuel with oxygen. This process involves the transformation of chemical energy to electrical energy, and back again.

Photosynthesis also involves the conversion of chemical energy to electrical energy, and back.

Changes in the electrical potential difference across the membranes of nerve and muscle cells are involved in the transmission of signals within a living organism.

The importance of electrical processes in respiration and nutrition has been understood for scarcely 20 years, while we have understood the electrical processes involved in nerve transmission for perhaps 40 years. These processes will be studied by life science students in great detail in a biological or chemical context. Of particular importance in these processes is the diffusion of charged particles, ions, through the membranes of living cells. In this chapter we study the underlying physics of this process, using relatively simple systems. This will form the basis of your later studies of electro-diffusive phenomena in the context of botany, physiology and other disciplines.

## 4-3 DIFFUSION OF IONS THROUGH A SEMIPERMEABLE MEMBRANE

The *diffusion of ions* refers to the tendency of charged fragments of molecules (ions) to move from regions of high concentration to regions of lower concentration. The diffusion is a direct result of the random thermal motion of the ions. (See chapter FE4.) The rate at which a particular ion species diffuses depends on many factors, including the average speed of the ion, which in turn depends inversely on its mass. Thus, in general, different ions will diffuse at different rates. Consequences include the following.

- The motion of the charged particles constitutes a net electric current .
- There will be a net separation of charge, establishing an electric field and corresponding potential differences. This field influences the transfer of ions in important and subtle ways.

In life, most electro-diffusive phenomena involve many different ion species and are, as a consequence, very complicated to analyse. However the principles can be studied using a simple system consisting of two solutions of a simple inorganic salt with different concentrations separated a by a membrane that lets through one kind of ion but not the other. (See the box on page 45.) Experiments show that a potential difference is set up between two solutions provided that

- the membrane is semipermeable (i.e. permeable to some ion species only) and
- the two solutions have different concentrations of ions.

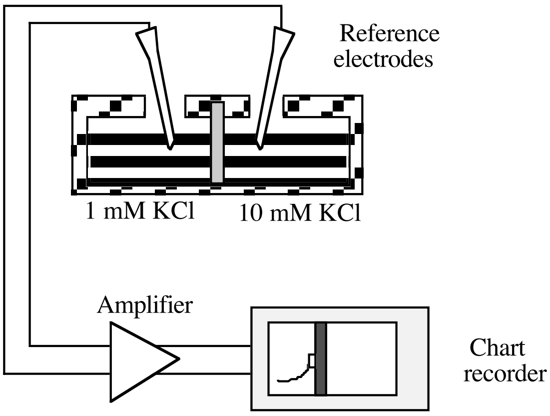
The potential difference between the two solutions very rapidly reaches an equilibrium value which is proportional to the logarithm of the concentration ratio.

The effect can be explained in terms of diffusion of ions through the membrane. Suppose that the membrane is permeable to the positive ions present but not to the negative ions. The ions which can get through are said to be **permeant**. All the ions, both positive and negative, are in rapid random thermal motion, continually colliding with each other, the solvent molecules, the walls of the container and the membrane. Although the motion is random more ions strike the membrane on the high concentration side than the other side simply because there are more of them. Some, but by no means all, of the positive ions reaching the membrane from the high concentration side start to work their way through, still bouncing around randomly as they go. Some get right through and some drift back. Fewer positive ions work their way through from the other side. Initially there are more permeant ions going from the high to the low concentration side than the other way. But not for long. The diffusion process soon creates an imbalance of charge on the two sides of the membrane. There is extra positive charge on the low concentration side, produced by the positive ions that got through, and an excess of negative charge on the high concentration side

## Demonstrations - Ion diffusion

### Apparatus

For the demonstration we have selected the simplest system we could find, namely two solutions of potassium chloride (KCl) in water, separated by a membrane. The apparatus consists of a plastic vessel divided into two compartments by a disk of membrane. The junctions between the two compartments and the membrane are sealed with rubber o-rings, and the whole is clamped to ensure that there is no leakage between the compartments. The only communication between the solutions in the two compartments is via the membrane.



The membrane is a disk of a semipermeable material called Permplex C-20 which is used in some types of water softener. It looks and feels like a piece of thin leather. It is permeable to potassium ions, but impermeable to chloride ions.

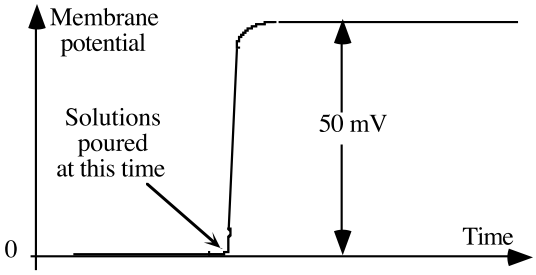
Aqueous solutions of potassium chloride are placed in the two compartments. The potential difference between the solutions is sensed by means of two **reference electrodes**, one in each compartment. The PD is amplified and recorded on an X-Y plotter.

Such reference electrodes are widely used in biological work to measure the potentials of solutions. Briefly, they are designed to make electrical contact with a solution without introducing any contact potential of their own. This rather complex but important question is discussed in chapter E5.

### Demonstration of diffusion potential

With the membrane and electrodes in place, solutions of KCl are poured into the two compartments: 10 millimolar on the right and 1 millimolar on the left.

As the solutions are poured, the potential difference between the compartments rises almost instantaneously to around 50 mV and then remains substantially constant. The left-hand (dilute) solution is positive relative to the right-hand (more concentrated) solution.



The result is consistent with the diffusion of positively charged potassium ions through the semipermeable membrane, producing a net transport of positive charge from the more concentrated to the more dilute solution. The membrane is impermeable to the negative chloride ions. There is therefore no compensating flow of chloride in the reverse direction, and a potential difference builds up across the membrane, the dilute solution being positive with respect to the concentrated solution.

### No effect with filter paper

The potential observed depends on the peculiar properties of the artificial membrane. When the membrane is replaced with filter paper, which is not semipermeable, and the experiment is repeated, the potential difference between the cells remains essentially zero.

Further experiments with the same apparatus show that the equilibrium diffusion potential of an ion depends on the concentration ratio of the two solutions and is proportional to the logarithm of that ratio.

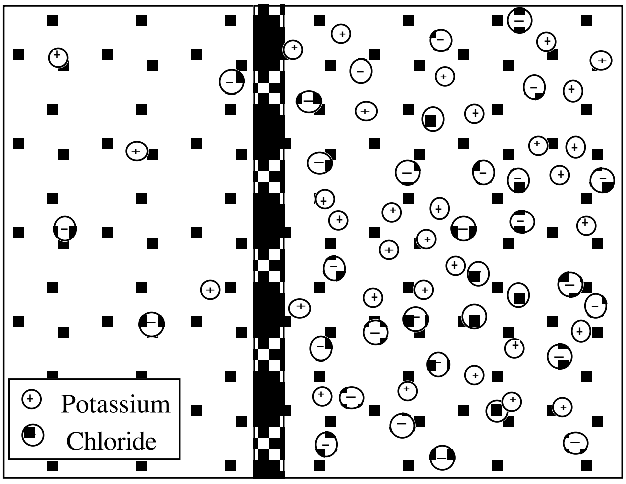
## A model for ion diffusion

A series of computer animations represent the motion of the ions in slow motion The ions move about randomly within the liquid. Their speeds and the directions of motion are continually changing as a consequence of collisions with the solvent molecules (which are not shown directly). Their motion is very similar to that of gas molecules.

### Totally impermeable membrane

A solid impermeable wall separates two solutions of different concentrations. The solution on the right-hand (RH) side of the wall has ten times the concentration of that on the left hand (LH) side. The average speed of the ions (and hence the temperature) and the average distance that the ions move between collisions with the solvent molecules are the same on both sides of the wall. The number densities of the (invisible) solvent molecules on both sides of the wall are also the same.

Although the ions move about randomly, they do not pass through the wall, but bounce back into the solution should they collide with it. Their concentrations remain unchanged. Note that more ions strike the RH side of the wall - i.e. the side on which the strong solution is located - than the other side.



### Semipermeable membrane

Now consider a membrane that is permeable to potassium ions, but impermeable to chloride ions. When a potassium ion strikes the wall it can move through. As before, chloride ions which strike the wall bounce back - they always remain on the same side of the wall.

Because the number density of potassium ions is greater on the right they strike the wall more often, so more potassium ions pass through from right to left than vice versa. Hence there is a net flow of potassium ions from the region of high concentration to the region of low concentration.

We have presented a simplified picture. Not every ion which strikes the membrane passes through it; ions certainly do not move through the membrane at a constant speed; and we have said nothing about the mechanism of diffusion through the membrane. Those matters are related to the detailed structure of the system: the computer animations illustrate only the results of diffusion through a membrane.

due to the negative ions which lost their positive partners. As a consequence of the attractive electrostatic forces between these opposite charges, the ions form layers near the surfaces of the membrane (figure 4.1). We have a charge double layer which creates an electric field inside the membrane, but not outside.

Now think what happens to a positive ion which might be in the process of diffusing through. As soon as it gets inside the membrane it experiences an electrostatic force pulling it back to the high concentration side. The two effects, diffusion sending ions one way and an electric field pulling them back, rapidly reach a balance - a dynamic equilibrium. The situation can be represented by saying that the diffusion current, which is determined by the temperature of the system, is balanced by a field current (figure 4.1).

Associated with the electric field in the membrane there is a potential difference across it, causing the whole of the low concentration solution to be at a higher potential than the high concentration solution. (See chapter E2 for the properties of charge double layers.)

The build up of electric field in the membrane and the potential difference across it are self-limiting. The direction of the field is such as to oppose the diffusion of those very ions which established it. Provided that no current is drawn by the measuring electrodes, the membrane potential reaches just that value required to reduce the net current through the membrane to zero.

The original concentrations of the permeant ions hardly change, as only a minute fraction of the ions present need to pass through the membrane in order to set up an equilibrium potential difference. This claim is consistent with the experimental observation of a rapid increase in potential to a constant value.

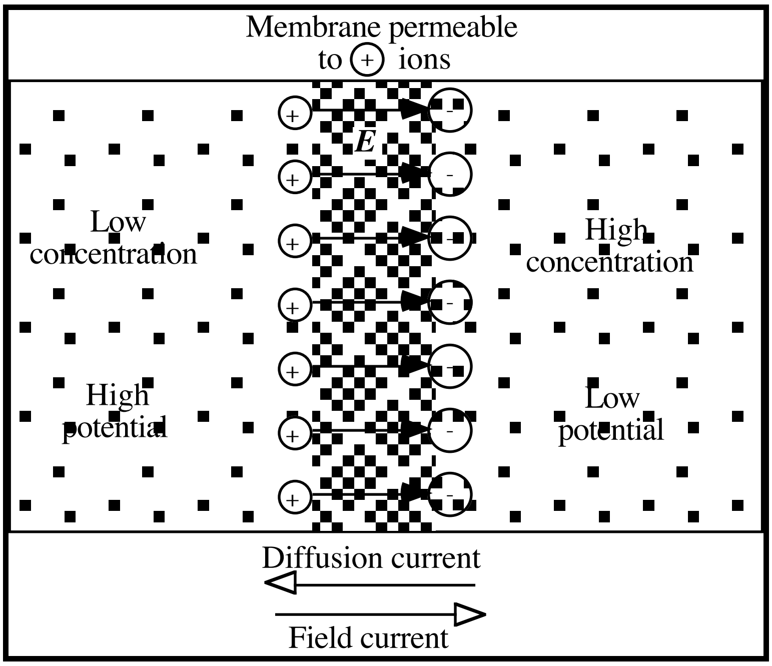


**Figure 4.1 The field inside a semipermeable membrane**

Here the membrane is permeable to the positive ions only. The electric field is set up in the membrane by diffusion of positive ions into the more dilute solution on the left.

**More demonstrations- Ion diffusion**

Experiments establish that the potential is not the result of some supposed asymmetry in the membrane: it is indeed a result of the concentration difference of the two solutions.

We repeat the diffusion experiment, after carefully washing the apparatus, with the semipermeable membrane and potassium chloride solutions of equal concentration (1 mM) in each compartment. No diffusion potential is observed. Then crystals of KCl are shaken into the right-hand compartment, which is stirred. The left-hand compartment (more dilute) immediately becomes positively charged, and the potential increases as the crystals dissolve.

A 1 mM solution of KCl is poured into the left-hand cell, and a series of solutions of strength 1, 2, 4, 8, 16 and 32 mM on the other side of the cell, the cell and membrane being carefully washed in distilled water between each measurement to minimise contamination.

Each time the concentration of potassium ions was *doubled* the potential across the membrane *increased* by the same voltage (18 mV). This indicates that the potential is proportional to the *logarithm* of the concentration of KCl in the RH cell. Further measurements in which the concentration of ions in the left hand cell was also changed would show that the *potential across the membrane is proportional to the logarithm of the concentration ratio*. Experimentally

$$V = \log_{10}\left(\frac{C_1}{C_2}\right) \times 60\ \text{mV}$$

where $C_1$ and $C_2$ are the concentrations on opposite sides of the membrane.

For $C_1/C_2 = 2.0$ this equation gives $V = 18$ mV. The constant factor (60 mV in this experiment) in the expression for the Nernst equilibrium potential of an ion depends on the ionic charge and the temperature, as discussed in the text.

## 4-4 NERNST EQUILIBRIUM

When the membrane is permeable to only one kind of ion and the steady potential difference is reached, the permeant ions are said to be in **Nernst equilibrium**. However the ions which can't get through the membrane are not in equilibrium. If they could get into the membrane the field would push them through to the low concentration side.

### Nernst equation

The equation for the **Nernst potential** is usually written using natural logarithms rather than base-10 logs.

$$\Delta V = \frac{kT}{ze} \ln\left(\frac{C_1}{C_2}\right) . \qquad \text{.... (4.1)}$$

Here $C_1$ and $C_2$ are the concentrations on opposite sides of the membrane, $k$ is the Boltzmann constant , $T$ is the absolute temperature, $z$ is the degree of ionisation of the permeant ion (usually 1 or 2) and $e$ is the electronic charge. This is the only form of the equation that you need to know.

The equation can also be written in terms of number densities, which are proportional to the concentrations:

$$\Delta V = \frac{kT}{ze} \ln\left(\frac{n_1}{n_2}\right) . \qquad \text{... (4.1a)}$$

***Note***

It is not worth trying to remember how to order concentrations and apply the sign of $z$ in equation 4.1. The **sense** of the electro-diffusive potential is more easily determined by considering which sign of ion diffuses through the membrane.

## 4-5 MEASUREMENT OF pH

One convenient and elegant way of determining the pH of a solution is to measure the Nernst potential across a membrane which is permeable only to hydrogen ions. The voltage across this membrane is then proportional to the logarithm of the ratio of the hydrogen ion concentrations on either side of this membrane.

Since $\Delta V \quad \log\left(\frac{C_1}{C_2}\right)$

and $\text{pH} \quad \log\left(\frac{C_\text{H}}{\text{mol.L}^{-1}}\right)$

then $\Delta V = \text{const} \times (\text{pH}_1 - \text{pH}_2)$ .

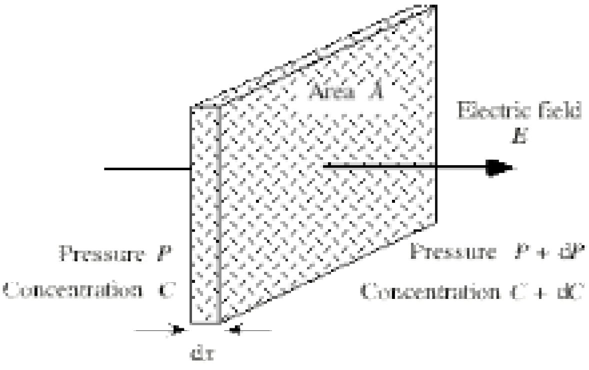
Thus if the pH of a standard solution on one side of the membrane is known, measurement of the membrane potential (the Nernst potential for $H^+$) gives a direct measure of the pH of the unknown solution on the other side of the membrane.

Thus pH can be directly measured with a high-resistance (ideal) voltmeter. This is the basis of operation of the pH meter shown in the video lecture. In this instrument and most other commercial pH meters, the ‘membrane’ is a specially compounded glass which is permeable only to $H^+$ ions. (See the box on page 50.)

### A Theory of the Nernst equilibrium

In this theory we regard Nernst equilibrium as the balance between *electrostatic force* on a layer of charge, and the *difference in the forces* exerted by the ions in collisions with the surfaces of that layer. We assume that the motion of the ions is similar to that of the molecules of a gas. This is a valid assumption provided that the concentration of the solute is not too great. (This presentation is slightly different from that given in the video lecture. The main difference is the replacement of the symbol $c$ (concentration) by $n$ (number destiny) in the early part of the derivation. The final result is identical.)

Suppose that the Nernst equilibrium has already occurred. Consider a thin layer of positive ions that are within the membrane, moving through. All those ions will feel an electrostatic force to the right from the electric field within the membrane. However this layer does not move, as the pressure of the ions on the right hand side of the layer is greater than that on the left. This pressure difference arises simply because there is a greater concentration of ions at the right of the layer. In a Nernst equilibrium the electric force is balanced by the force resulting from the pressure difference on the layer and there is no net flow of ions into, or out of, the layer.



For the layer, this equilibrium condition can be expressed as the equation:

(Magnitude of electric field) × (total charge) = force from pressure difference ;

$$E.\,nze.A\,\mathrm{d}x \quad = \quad A\,\mathrm{d}P$$

Hence, by cancelling the area $A$, we get $\quad E.nze.\mathrm{d}x = \mathrm{d}P \qquad$ .... (1)

where $n$ is number density of ions, $z$ is the ionisation state of the ion, and $e$ is the electronic charge.

Assuming that the ions obey the ideal gas law (detailed investigation shows this to be true for dilute solutions), $\quad P = nkT \;$ or $\; P = CRT \qquad$ ... (2)

where $k$ is the Boltzmann constant ($R/N_0$), $T$ is the temperature, $R$ is the gas constant and $N_0$ is the Avogadro constant.

From equation 2 we get $\quad \mathrm{d}P = kT.\mathrm{d}n \;$ or $\; \mathrm{d}P = RT\mathrm{d}C$ .

This, together with the relations $\quad n = N_0C \,; \quad k = R/N_0$

allows us to rewrite equation 1 as $\quad E.Cze.\mathrm{d}x = kT.\mathrm{d}C \qquad$ .... (3)

or $\quad E\,\mathrm{d}x = \mathrm{d}V = \dfrac{kT}{ze}\dfrac{\mathrm{d}C}{C}$

Integration through the thickness of the membrane gives the Nernst equation:

$$\Delta V \quad = \quad \frac{kT}{ze}\ \ln\left(\frac{C_1}{C_2}\right)$$

where $\Delta V = \int_{x_1}^{x_2} E\,\mathrm{d}x$ is the potential difference across the membrane, and $C_1$ and $C_2$ are the concentrations on opposite sides of the membrane.

Expressed with different constants and base-10 logarithms the equation takes the form

$$\Delta V \quad = \quad 2.303\,\frac{RT}{zF}\ \log_{10}\left(\frac{C_1}{C_2}\right) \qquad \text{...(4)}$$

where $F$ is the Faraday constant (see chapter E3).

You should satisfy yourself that the constant factor is indeed about 60 mV for a singly ionised ion at room temperature, as found experimentally.

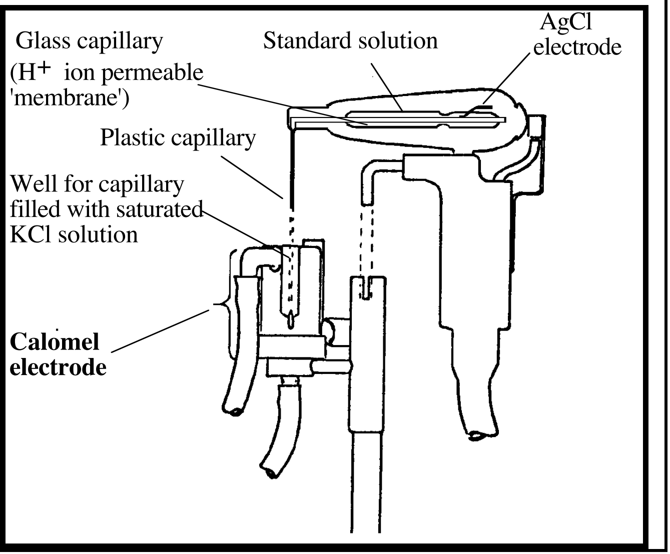
**Demonstration - pH meter**

A pH meter is used to check the pH of the blood of a patient recovering from anaesthesia. The pH meter shown is specifically designed for a small sample of blood. The glass membrane is a capillary tube sealed into a glass tube filled with a liquid of known, standard pH.

The head is separated from its stand to draw a blood sample up the plastic capillary to fill the glass capillary, which is permeable to $H^+$ ions. (The stand and head are shown separated in this diagram.) The standard reference solution is in the glass tube surrounding the glass capillary.

When the head is returned to the stand the blood in the plastic capillary makes electrical contact with the saturated KCl solution which is part of the calomel reference electrode.

The reference electrodes used in this instrument are designed so as to minimise the spurious potentials which arise at the various liquid-liquid interfaces. (For a discussion of bioelectrodes such as the calomel electrode see chapter E5.)



## 4-6 ION DIFFUSION IN BIOLOGY

We have so far discussed in detail only simple systems in which the potential across a membrane was determined by the diffusion of *only one* ion species through the membrane, the membrane being impermeable to other ions. In that case the system rapidly reaches an equilibrium, and the potential is just equal to the Nernst potential for that ion.

In more complicated systems, notably living cells where many types of ions are involved, the observed potential is not necessarily equal to the Nernst potential of any particular species of ion. When a membrane potential does not correspond to the Nernst potential for some kind of ions there will be a net flow of those ions through the cell membrane. In the long term, such departures from equilibrium can exist only if ions are transported through a cell membrane by some active mechanism which transports diffused ions back where they came from.

Active metabolic processes in the wall of a living cell perform that function. Two examples where such biological processes are important in determining membrane potentials are nerve potentials and plant cell metabolism.

### Nerve and muscle potentials

An electric potential difference of about 90 mV across the surface membrane of the nerve and muscle cells in animals has an important role in the electrical signalling within the organism. The sodium ion concentration outside a nerve cell is greater than that inside. (See figure 4.2). So sodium ions diffuse into the cell. Since the inside of the cell is negative with respect to the outside any sodium ions which get into the membrane will also be pushed into the cell by the field in the membrane. These effects add to give a net inward flow of sodium. To maintain the concentration difference, then, sodium ions must be actively removed from the cell. The **sodium pump** which balances the influx of sodium ions is a protein which expends metabolic energy in transferring the ions out of the cell.

As the potassium ion concentration is much greater inside the nerve cell than outside, diffusion will produce an outward flow of potassium ions, whilst the electric field will encourage these ions to move in to the cell. The potassium ions are nearly, but not quite, in Nernst equilibrium. A **potassium pump** maintains this slight departure from equilibrium. The potassium pump does not have to expend nearly as much energy as the sodium pump which is maintaining a gross departure from equilibrium.

The cell potential and the chloride ion concentration are such that there is no net flow of chloride ions through the cell wall - the chloride ions are in Nernst equilibrium.

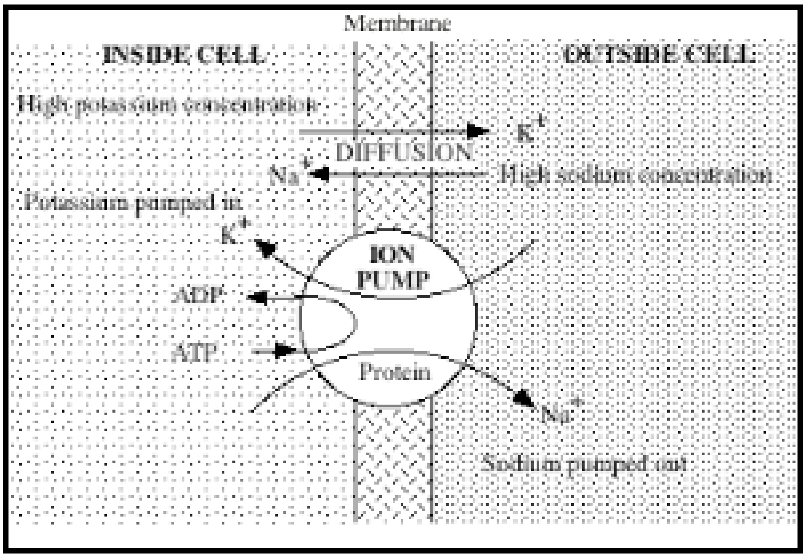


**Figure 4.2 Conceptual model of ion transport in nerve cells**

**Metabolism in plant cells**

In many cells, such as those of plants, fungi and bacteria, the potential across the cell wall is determined directly by proteins which expend chemical energy to move ions through the cell (see figure 4.3). Those proteins move hydrogen ions *outwards* through the membrane. The ions return through other proteins which acquire the sugar and other foods required for plant nutrition. Thus there is a continual circulation of hydrogen ions through the membrane in the process of food gathering by the cell. Here the electric potential difference cannot be understood in terms of the diffusion of ions; it is caused directly by the movement of hydrogen ions as a consequence of chemical processes. (See figure 4.3.)

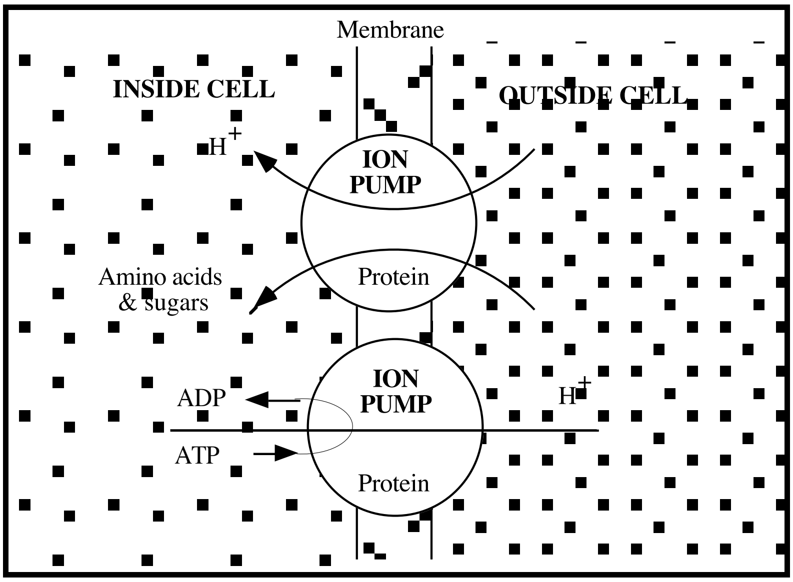


**Figure 4.3 Ion and food transport in plant cells.**

## POST-LECTURE

### 4-7 MEMBRANE POTENTIALS AND DONNAN EQUILIBRIUM

We have already seen that

a) ion diffusion plays a central role in the life process of most cells and

b) the phenomena are extremely complex as many ion species are involved, with different concentrations and permeabilities.

In this section we discuss some of the basic physics determining the evolution and equilibrium of non-living systems. Although we are discussing these matters qualitatively, the principles are widely used by biophysicists in setting up the mathematical relationships which enable them to understand the electrodiffusive properties of living cells.

Suppose we have two compartments containing solutions of various ions in various concentrations, separated by a membrane which is permeable in differing degree to the various ions. To be quite general, there need be no membrane at all, just a liquid-liquid junction (such as that mentioned between the blood sample and the saturated potassium chloride (KCl) solution in the description of the blood pH apparatus). For the moment we assume no external electric connection between the compartments, and no active biological mechanism in operation. What then can physics tell us about this system?

Two basic principles apply (based on considerations discussed in chapters E1, E2 and E3). They are as follows.

**1. Bulk charge neutrality**

The bulk solutions on each side of the boundary must both be essentially electrically neutral, that is each solution must contain equal amounts of positive and negative ionic charges. There may of course be a surface charge - a charge double layer - but that exists only on the surfaces of the membrane. In symbols

$$\sum_i n_i q_i = 0$$

on each side of the membrane.

## 2. Zero membrane current density

As ions can diffuse through the membrane, there must be zero mean current density through the membrane if bulk charge neutrality is to be maintained. There is no completed circuit for the current, and a continuing separation of charge would violate bulk charge neutrality. Of course there is a very small, transient current as any charge double layer is established or changed. In symbols

$$\sum_{i} n_i q \bar{v}_i = 0$$

where $\bar{v}_i$ is the component of mean velocity, along an axis perpendicular to the membrane, of ions of species *i*.

In the KCl example discussed in the lecture, $K^+$ was the only ion which could pass through the membrane, so zero net current also meant zero net ion flow, and an equilibrium was quickly established. In the general case, when there is more than one kind of permeant ion, there is no such equilibrium. Ions can (and do) diffuse both ways through the boundary, subject only to the constraint of zero mean current density. Roughly this means that for every positive ion that diffuses to the left, *either* a negative ion also diffuses to the left, *or* another positive ion diffuses to the right.

If the different ions have different concentration ratios it is likely that the value of the Nernst potential will be different for each kind of ion. It is therefore impossible for all the ions to be in Nernst equilibrium, because there can be only one value of the potential difference across the membrane. One species of ion at most could be in equilibrium, but it is more likely that none are. Since there is no equilibrium the system must keep changing.

The rates at which the various ions diffuse are determined by the permeability of the membrane (and/or the mobility of the ions in the solutions). A **junction potential** (alias liquid junction potential, diffusion potential, membrane potential, boundary potential etc.) is usually set up by this ion diffusion. The sign and magnitude of the potential are determined by the concentrations and permeabilities of all the ion species. The most permeant ions are, understandably, dominant in determining the potential. (This point is of vital importance in understanding the propagated action potential of nerve cells). The junction potential influences the flow of the various ions, but unlike our single-ion example, it does not stop the flow of any particular ion. The potential builds up rapidly to just that value required to ensure that the mean current density carried by all ions is zero.

But that situation is not an equilibrium. As time goes on, the concentrations of the ions and the junction potential evolve gradually towards a final equilibrium state. The time scale for this evolution may be days or months for laboratory-scale apparatus but for typical living cells would be minutes or less - if it were allowed to happen.

The course of this evolution is controlled by the concentrations, the permeabilities and the zero current principle. The end point of the evolution, the ultimate equilibrium state, is governed by the principle of bulk charge neutrality.

To see how this works, suppose that among all the various ions, there is just one species to which the membrane is totally impermeant. To be definite, suppose that the species is negatively charged and exists only in the RH compartment of the system in figure 4.4. This system will evolve as discussed above. It is clear that in the final state there must be an excess of permeant positive ions (over permeant negative ions) in the RH compartment, if bulk neutrality is to be maintained. As a consequence there must be a potential across the membrane, with the left hand side positive, to stop these ions diffusing through the membrane.

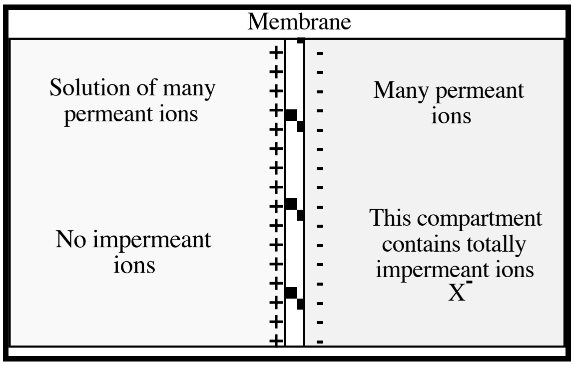


**Figure 4.4 Donnan equilibrium.**
A vessel is divided into two compartments by a semipermeable membrane. Many ions species are present, but only the right hand compartment contains the totally impermeant ions $X^-$. The system evolves to a Donnan equilibrium. The totally impermeant $X^-$ ions in the right hand compartment maintain a junction potential, with the left hand side positive. All other ions have reached Nernst equilibrium.

This end-point equilibrium must eventually be reached by all liquid-liquid interfaces unless some other processes intervene. It is called a **Donnan equilibrium**, and the potential difference is called the **Donnan potential**.

A Donnan equilibrium has two characteristic features by which it can be recognised.

1. All ion species which are permeant (i.e. able to pass through the boundary) are in Nernst equilibrium at the final junction potential, i.e. their concentration ratios have adjusted to satisfy the Nernst equation, where the potential in the Nernst equation is the Donnan potential.
2. The only ions not in Nernst equilibrium are totally impermeant; they can never pass through the boundary.

If the original solutions contain *no* totally impermeant ions, then the concentrations and the junction potential evolve until all ions are equally distributed between the two compartments, and the junction potential is zero. This is a special (degenerate) case of Donnan equilibrium: all the ions are in Nernst equilibrium at a junction potential of zero. For each ion the concentration ratio is 1:1.

Thus, physical arguments show that two liquids separated by a membrane permeable to *some* of the ion species present will evolve to an equilibrium sate - a Donnan equilibrium - in which all the **permeant ions** are in Nernst equilibrium.

## 4-8 MEMBRANE POTENTIALS OF LIVING CELLS

Now most living cells have been shown experimentally *not* to be in Donnan equilibrium. The reason for this surprising result, which puzzled physiologists early this century, is that the concentrations are not determined by diffusion through the cell membrane, but that other active cell processes (such as the so-called sodium pump) are at work maintaining the non-equilibrium concentrations. These cells are maintained in this steady state by cell metabolism, which actively transports ions from a region of low concentration, to a region of high concentration. In such cases the potential of the living cell can be described as a ‘steady state liquid junction potential’.

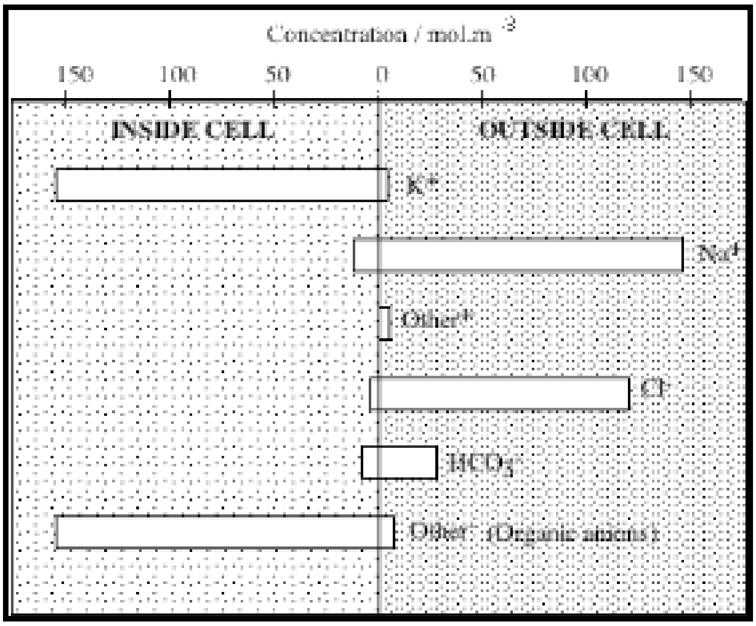


**Figure 4.5 Ion concentrations for mammalian muscle cells**

Some values of ion concentrations for typical mammalian muscle cells are shown in table 4.1 and figure 4.5. It is interesting to enquire whether any of the ions are in Nernst equilibrium. The resting potential of the cell is known from experiment to be close to 90 mV. The Nernst potential for each kind of ion can be calculated from the Nernst equation, as shown in the following example.

| Fluid **outside** the cell | | | Fluid **inside** the cell | | |
|---|---|---|---|---|---|
| **Ion** | | $C$ / mol.m$^{-3}$ | **Ion** | $C$ / mol.m$^{-3}$ | Nernst potential $V$ / mV |
| potassium | K$^+$ | 4 | K$^+$ | 155 | -97 |
| sodium | Na$^+$ | 145 | Na$^+$ | 12 | 67 |
| other cations | X$^+$ | 5 | | | |
| chloride | Cl$^-$ | 120 | Cl$^-$ | 4 | -90 |
| bicarbonate | HCO$_3^-$ | 27 | HCO$_3^-$ | 8 | -32 |
| other anions | | 7 | organic anions | 155 | |

**Table 4.1 Ion concentrations for mammalian muscle cells**

**Example: Calculation of Nernst potential**

To calculate the Nernst potentials for the ions in table 4.1, it pays to split the calculation into two parts. The first part is to calculate the constant in front of the log, because that will be the same for all singly charged ions. (If we had any doubly charged ions, their value of the constant would be just half that for the single charge.) We need the temperature: 37°C or 310 K is normal for humans. We also need

the Boltzmann constant $k = 1.38 \times 10^{-23}$ J.K$^{-1}$.

and the electronic charge $e = 1.60 \times 10^{-19}$ C.

So
$$\frac{kT}{ze} = \frac{(1.38 \times 10^{-23}\ \mathrm{J.K^{-1}}) \times (310\ \mathrm{K})}{1 \times (1.60 \times 10^{-19}\ \mathrm{C})}$$

$$= 26.7 \times 10^{-3}\ \mathrm{J.C^{-1}} = 26.7\ \mathrm{mV}.$$

Now for the sodium ($Na^+$) ions the concentration ratio is 145/12 so

$$\Delta V = \frac{kT}{ze} \ln \left(\frac{C_1}{C_2}\right)$$

$$\Delta V = (26.7\ \mathrm{mV}) \times \ln(145/12) = 67\ \mathrm{mV}.$$

To sort out the polarity we note that the higher concentration is outside the cell, so positive ions diffuse in, giving the inside a positive charge and therefore a potential higher than the outside. Taking the outside fluid as having zero potential, the usual convention in cell physiology, the sodium Nernst potential for the inside of the cell is + 67 mV. The sodium is clearly not in Nernst equilibrium because the inside of the cell is actually at a potential of - 90 mV. Even the sign is wrong!

If you compare the Nernst potentials listed in table 4.1 with the actual value of the internal cell potential, -90 mV with respect to the outside, you can see that only the chloride ions are in Nernst equilibrium. Sodium is far from equilibrium, having the wrong sign for its potential, while potassium is not too far from equilibrium. There must be an active biological process, the sodium-potassium pump mentioned in the lecture, that maintains the permeant ions in their non-equilibrium concentrations.

## 4-9 QUESTIONS

### Data

Molar masses: K: 39.1 g mol$^{-1}$, Cl: 35.5 g.mol$^{-1}$, Li: 6.9 g.mol$^{-1}$, Na: 23.0 g.mol$^{-1}$.

**Q4.1** Calculate the mass of potassium chloride needed to make up 1.00 L of 5.00 millimolar (concentration 5.00 mmol.L$^{-1}$) solution.

**Q4.2** What volume of solution is obtained if water is added to 20 mg of sucrose (ordinary sugar, molar mass = 342 g.mol$^{-1}$) to make up a 5 mM solution?

**Q4.3** The fluid in a particular class of nerve cells contains lithium ions in solution with a concentration of 1.0 $\mu$mol.L$^{-1}$.

a) What mass of lithium will be in solution in such a cell when the volume of the cell is $1.0 \times 10^{-9}$ m$^3$?

b) How many ions does this correspond to?

**Q4.4** A membrane of 1.0 mm thickness divides a cylindrical vessel into two compartments, each 10 mm long by 20 mm diameter. Solutions of different concentrations of NaF are poured into these compartments, giving rise to a potential of 100 mV between them. The membrane is permeable only to $Na^+$ ions and has a relative permittivity of 2.0.

a) Calculate the electric field within the membrane.

b) Calculate the charge density and the *total* charge on each surface of the membrane.

c) Hence calculate the *fraction* of the $Na^+$ ions present in the stronger solution which have diffused through the membrane, given that the concentration of this solution is 10 mmol.L$^{-1}$.

**Q4.5** An artificial membrane divides a cylindrical vessel into two compartments, into which are poured solutions of concentration 1.00 mmol.L$^{-1}$ and 5.00 mmol.L$^{-1}$. The membrane is permeant to $K^+$ ions, but not $Cl^-$ ions.

a) What potential will appear between the compartments? Which cell will be at the higher potential?

b) How will this potential change if the concentration of the weaker solution is increased to 2.00 mmol.L$^{-1}$?

The apparatus is at room temperature (293 K).

**Q4.6** The membrane in the apparatus described in question 4.5 is replaced with a membrane permeant to $Na^+$ ions, but not $Cl^-$ ions. A potential of 100 mV appears across the membrane when NaCl solutions of different concentrations are poured into the compartments. The concentration of the solution in one compartment is 5.00 mmol.L$^{-1}$, and this compartment is *positive* with respect to the other. What is the concentration of the solution in the other compartment? The temperature is 293 K.

**Q4.7** The membrane in the apparatus described in question 4.5 is replaced by a membrane permeant to $Ba^{2+}$ ions, but not to $SO_4^{2-}$ ions. A potential of 50 mV appears across the cell when $BaSO_4$ solutions of different concentrations are poured into the compartments. The concentration of the solution in one compartment is 10.0 mmol.L$^{-1}$, and this is *negative* with respect to the other. What is the concentration of the solution in the other compartment?

**Q4.8** An artificial membrane is permeant to $Na^+$, $K^+$, $Cl^-$ and $Br^-$ ions, but *not* to a complex uranyl ion. The uranyl ion is present in different concentrations on opposite sides of the membrane, and hence gives rise to a Donnan potential across the membrane.

The concentrations of the permeant ions on one side of this membrane when in Donnan equilibrium are as follows.

| Ion | Concentration/mmol.L$^{-1}$ |
|---|---|
| $K^+$ | 0.100 |
| $Na^+$ | 1.00 |
| $Cl^-$ | 2.00 |
| $Br^-$ | 0.0100 |

The concentration of the $K^+$ ion on the other side of this membrane is 2.00 mmol.L$^{-1}$.

a) What is the Donnan potential across the membrane?

b) Calculate the concentration of the $Na^+$, $Cl^-$ and $Br^-$ ions on the other side of the membrane.

Assume that the membrane and solutions are at room temperature.

**Hint:** What do you know about the concentrations of permeant ions in a Donnan equilibrium?

# E5 ELECTRODES AND ELECTROLYTES

## OBJECTIVES

### Aims

From this chapter you should develop an understanding of the basic physical processes involved in electrodes and their applications. You should be able to describe the processes in words and apply the principles of ion diffusion to electrode processes. You should be able to discuss examples of various types of electrodes and electrolysis. After studying chapters E4 and E5 you should be able to discuss the importance of electrochemical science and the fundamental idea of Nernst equilibrium.

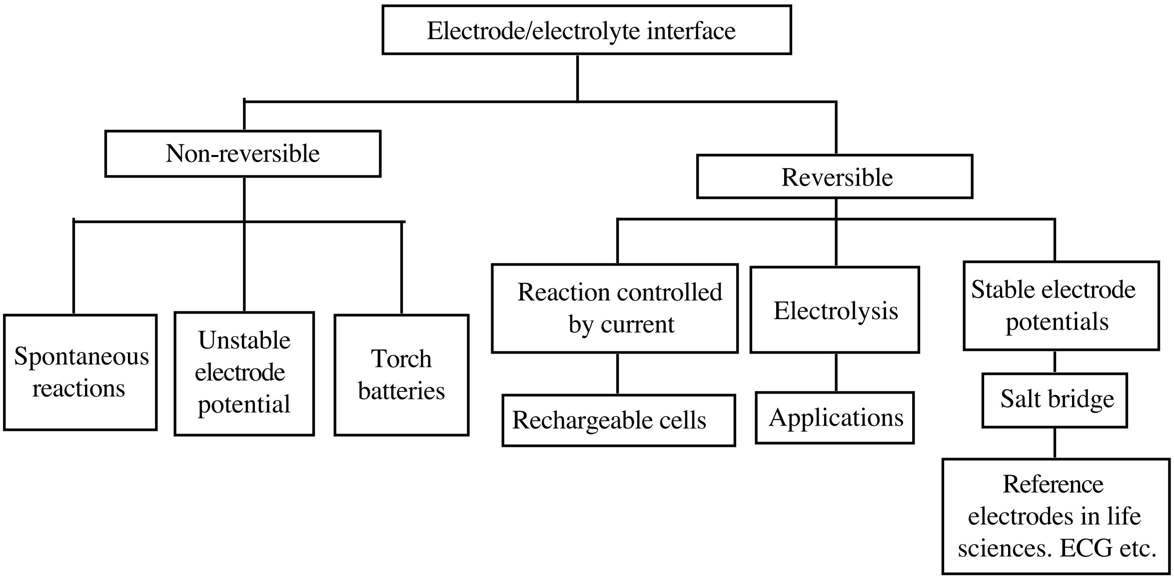
### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms:

   *electrolyte, electrode, galvanic cell, hydrogen reference electrode, electrode potential, cell potential, standard electrode potential, electrochemical reversibility, bioelectrode, battery, electrolysis, overpotential* .
2. Describe and explain the behaviour of reversible and irreversible metal-electrolyte electrodes and their electrode potentials in terms of ions, electrons and currents.
3. Describe and explain how cell potentials arise.
4. Describe the structure, function, properties, advantages and uses of various types of electrodes including bioelectrodes, reversible ECG electrodes, reference electrodes
5. Describe and explain the process of electrolysis including the distribution of electric potential within electrodes and electrolyte and mass transfer using an example such as the copper-copper sulfate-copper system.
6. Calculate masses deposited in electrolysis (see also chapter E3).
7. Describe applications of electrolysis.

## CONCEPT DIAGRAM



# PRE-LECTURE

## 5-1 THE INTERFACE BETWEEN MATERIALS

Chapter E4 was concerned primarily with the diffusion of ions through semipermeable membranes which separate solutions of different concentrations, and the electrical effects that resulted from that diffusion. In this chapter we extend those ideas to describe the electrical phenomena which result when dissimilar materials are placed in contact - in particular we discuss what happens when a metal electrode is placed in an ionic solution, but the ideas developed are equally applicable when solids with different properties are in contact, as in a semiconductor junction for example.

The boundary between the dissimilar materials is like a semipermeable membrane; *some* of the ion species present can diffuse from one material to the other. The electrical conditions at the interface are determined by the same basic conditions that we used when discussing membrane potentials, namely:

- bulk neutrality and
- zero current density if the external electric circuit is not complete.

Motion of ions through the interface is determined by these electrical conditions and by the possible chemical reactions which can occur at the interface. The chemical reactions play a role analogous to that of the permeabilities in determining the transfer of ions through semipermeable membranes.

For example, when copper is placed in a copper sulfate solution the chemical reaction which takes place at the surface of the copper is

$$\mathrm{Cu} \rightleftharpoons \mathrm{Cu}^{2+} + 2\mathrm{e}^- \ .$$

This *chemical equation* expresses the idea that a copper atom (Cu) can become a doubly charged copper ion ($\mathrm{Cu}^{2+}$) by giving up 2 electrons ($\mathrm{e}^-$), a change indicated by the right-pointing arrow. The left-pointing arrow indicates that the reaction can go the other way: a copper ion can pick up two electrons to become a neutral atom. In this reaction the only ion which can cross the interface is $\mathrm{Cu}^{2+}$. Consequently the zero current condition corresponds to a zero flow of ions across the interface, just as if the interface were a semipermeable membrane permeable only to $\mathrm{Cu}^{2+}$ ions. Hence a charge double layer builds up to give a Nernst equilibrium at the interface.

By contrast if magnesium is placed in acid the reaction is:

$$\mathrm{Mg} + 2\mathrm{H}^+ \rightleftharpoons \mathrm{H}_2 + \mathrm{Mg}^{2+};$$

a magnesium atom (Mg) in the metal combines with two hydrogen ions ($\mathrm{H}^+$) in the solution, releasing a molecule of hydrogen gas ($\mathrm{H}_2$) and leaving a doubly charged magnesium ion ($\mathrm{Mg}^{2+}$) in solution. In this case two species of ions can cross the interface. The interface potential mediates the reaction in the sense that it ensures that the currents carried by $\mathrm{H}^+$ and $\mathrm{Mg}^{2+}$ ions cancel out. The interface behaves like a membrane which has different permeabilities to $\mathrm{Mg}^{2+}$ and $\mathrm{H}^+$ ions.

## 5-2 EMF AND POTENTIAL DIFFERENCE

Chemical reactions between metals and electrolytes can be exploited to produce electrical energy from chemical energy. The chemical reactions are responsible for producing charge separation at the boundaries between solid metal and a solution containing its ions. We can describe this energy conversion process in terms of an abstract quantity called EMF (symbol $\mathcal{E}$). As noted in chapter E2 EMF can be defined as the energy per charge given to a system of charges. Once they are separated the charged particles create an electrostatic field and its associated potential difference. If you look at a static situation such as the separated charge on the + and - terminals of a battery, the PD between the terminals must be equal to the EMF which created it: $V = \mathcal{E}$. (That equality no longer holds if you extract energy from the battery by putting it in a circuit and letting current through it.) Since the quantity that is measured in practice is the potential difference, we shall discuss the effects of EMFs in terms of potentials.

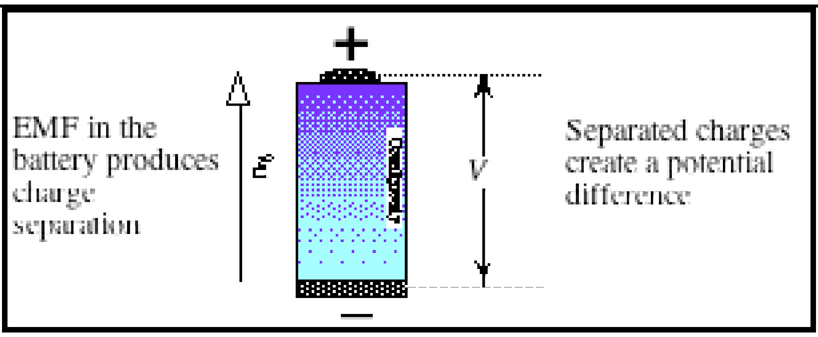


**Figure 5.1 Battery voltage**

## LECTURE & TEXT

## 5-3 WHAT ARE ELECTRODES?

*Electrodes are the means by which liquids and living organisms are included in electrical circuits.*

**Demonstrations**

- In a plant physiology lab the potentials within the cells of the fresh water plant *Chara* are measured by pushing a fine glass needle containing an ionic solution through the cell wall. This needle and the system by which it is connected to the wires which lead to the recording equipment is referred to as an **electrode**.
- The zinc outer case of a torch battery is the negative electrode of the battery, and the carbon rod down the centre of the cell is the positive electrode.
- In electrochemistry hydrogen gas bubbling over a sheet of platinum forms a **hydrogen reference electrode**.
- A basket of nickel pellets in an electroplating bath forms the positive electrode of the electroplating cell.

## 5-4 ELECTRODES AND ELECTROLYTES

### Electrode potentials

If you put strips of two different metals into an appropriate electrolyte solution you are likely to find that a potential difference appears between the two strips. The system is a **galvanic cell**, commonly known as a battery. The potential difference is produced by an EMF associated with the chemical reactions between both the metals and the electrolyte. The interface between each piece of metal and the electrolyte solution forms an **electrode**. Electrodes are always used in *pairs*. It is impossible to make voltage measurements or pass current into an electrolyte without using two electrodes.

A simple example is a piece of copper (Cu) and a piece of zinc (Zn) both partly immersed in the same dilute solution of copper sulfate ($CuSO_4$). The copper becomes about 1 volt positive with respect to the zinc. The potential change occurs in two steps at the *electrode-electrolyte interfaces*, not in the bulk solution. (See figure 5.2.) Both metals are negative relative to the electrolyte, but the potential of the zinc is more negative than that of the copper. So the copper is positive with respect to the zinc.

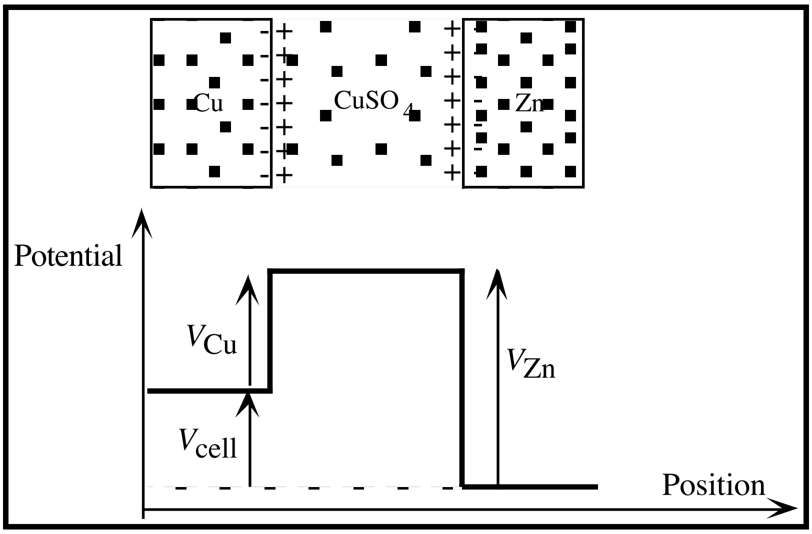


**Figure 5.2 Potential distribution in a cell on open circuit**
The cell potential is the difference between the potential changes at the two interfaces. Note that the actual values of the individual potentials $V_{Cu}$ and $V_{Zn}$ cannot be measured; only the difference $V_{cell} = (V_{Cu} - V_{Zn})$ can be found.

It is impossible to measure these absolute **electrode potentials** directly because you can't make contact with the electrolyte without using another electrode. If you introduce a third electrode the problem is still there because you don't know its electrode potential. However the **cell potential**, which is the difference between the two electrode potentials, can be measured directly.

### Metal - electrolyte electrode

The chemistry, electrode potential, and other characteristics of an electrode depend not only on the solid electrode material, but also on the solution in which it is placed. Strictly the *electrode* is not just the metal, but the combination of the metal and the solution. Thus for example, when copper is placed in a copper sulfate solution we refer to a *copper - copper sulfate* electrode.

### Nernst equilibrium potential

The chemistry of the electrode reaction is such that neither electrons in the metal nor negative ions in the solution cross the interface. The physical processes at the interface between the metal and a solution of its ion are similar to those that occur when a single ion diffuses through a semipermeable membrane. The tendency of metal ions to diffuse from the electrode to the electrolyte (i.e. from a strong 'solution' to a weaker solution) or the other way is balanced by the potential difference across the interface, creating a Nernst equilibrium.

The potential of a metal electrode relative to a solution of its own ion might be modelled using the *Nernst equation.*

$$V_{\text{metal}} - V_{\text{solution}} = \frac{kT}{ze}\ln\left(\frac{C_{\text{solution}}}{C_{\text{metal}}}\right)$$

where $C_{\text{solution}}$ is the concentration of metal ions in the electrolyte and $C_{\text{metal}}$ is the effective concentration of ions in the solid electrode. One trouble with this model is that we cannot accurately assign a theoretical value to $C_{\text{metal}}$, nor can it be measured. So the equation needs to be manipulated into a useable form. The first step is to introduce a standard reference value for concentrations, $C_{\text{ref}}$, and to split the right hand side of the equation into two terms:

$$V_{\text{metal}} - V_{\text{solution}} = \frac{kT}{ze}\ln\left(\frac{C_{\text{solution}}}{C_{\text{ref}}}\right) + \frac{kT}{ze}\ln\left(\frac{C_{\text{ref}}}{C_{\text{metal}}}\right)$$

$$= -\frac{kT}{ze}\ln\left(\frac{C_{\text{metal}}}{C_{\text{ref}}}\right) + \frac{kT}{ze}\ln\left(\frac{C_{\text{solution}}}{C_{\text{ref}}}\right)$$

Next, we hide the troublesome idea of the concentration of ions in the solid metal by labelling the first term as the **standard electrode potential**, $V_{\text{electrode}}$ and looking for a way of measuring it. The second term could be called a **concentration potential**. The equation is now

$$V_{\text{metal}} - V_{\text{solution}} = V_{\text{electrode}} + \frac{kT}{ze}\ln\left(\frac{C_{\text{solution}}}{C_{\text{ref}}}\right) \qquad \text{... (5.1)}$$

where we have written $V_{\text{electrode}} = -\frac{kT}{ze}\ln\left(\frac{C_{\text{metal}}}{C_{\text{ref}}}\right)$ - note the minus sign!

There are some important interpretations to be made about this equation. Firstly it says that there are two things that contribute to an electrode potential: (1) a contact between a metal and a solution of its own ion and (2) the concentration of that ion in the solution. We can now talk about those two contributions separately, but first we need to specify a value for the reference concentration. The universally accepted value is one mole per litre: $C_{\text{ref}} = 1$ mol.L$^{-1}$.

Now for the problem of measuring the standard electrode potential. We have already seen that it can't be measured directly, because you need another electrode which has its own, unknown, electrode potential. The problem is solved by carefully specifying a standard reference electrode, so that all other electrode potentials can be referred to it. The chosen standard is a device called the **standard hydrogen electrode**. (See the right hand side of figure 5.3.) The metal electrode consists of platinum metal coated with platinum black (colloidal platinum), which absorbs and holds gaseous hydrogen. The lower part is immersed in an exactly 1 molar solution of hydrogen ions ($H^+$) while the upper part is surrounded by hydrogen gas. Hydrogen gas is continually bubbled through the arrangement.

The remaining problem is how to introduce the reference electrode and make the electrical connections between the electrode you want to measure and the reference electrode. We need to connect two different electrolytes without introducing another electrode or any additional potentials. The usual techniques all involve connecting the two electrolyte solutions using something that allows ion flow without mixing the two main electrolytes. One method is to use a **salt bridge**, a thin tube filled with electrolyte; another is to place a porous wall between the two electrolytes.

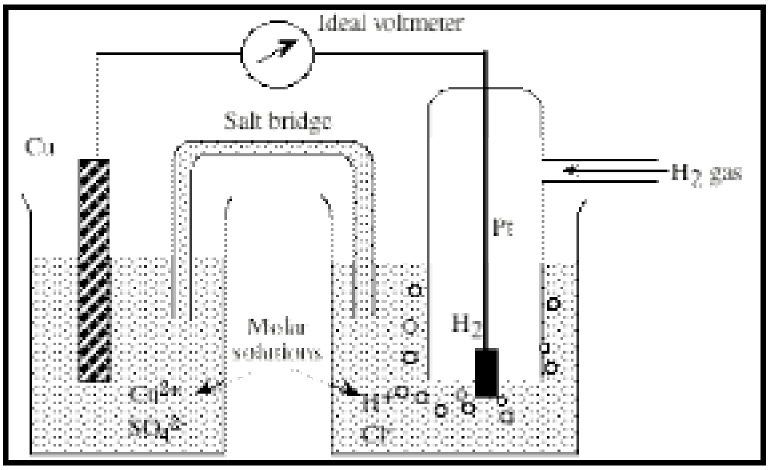


**Figure 5.3 Measurement of a standard electrode potential**
The electrode to be measured is in a molar solution of its own ion. That solution is electrically connected to a standard hydrogen electrode by a salt bridge. The concentrations of the metal ions in the left hand cell and the hydrogen ions on the right must both be equal to 1.00 mol.L$^{-1}$.

It is often said that the electrode potential of the hydrogen electrode is defined to be zero, but that is not strictly true because the potential difference between metal electrode and electrolyte is really unknown. It is better to say that all other electrode potentials are the potential difference that you get when you make them into a galvanic cell with a hydrogen electrode.

You don't always have to use a hydrogen electrode. Once the electrode potential of one metal has been measured relative to the hydrogen electrode, you can use that electrode as a reference for other electrodes.

The standard electrode potential could be either positive or negative, depending on the chemical reaction that occurs at the electrode when it is included in a cell with a hydrogen electrode. If atoms of metal X lose electrons and go into solution, the chemical reaction could be

$$X \rightarrow X^{+} + e^{-}$$

or

$$X \rightarrow X^{2+} + 2e^{-} \quad \text{etc.}$$

On the other hand ions might come out of solution:

$$X^{+} + e^{-} \rightarrow X \quad \text{etc.}$$

The reaction and the sign of the standard electrode potential depend on what happens when the electrode is made into a complete cell with a hydrogen electrode. Table 5.1 shows some examples. Although this table shows the reactions that occur when the electrode is part of a cell that includes a hydrogen electrode, they can go the other way in other arrangements. The directions of the reactions are determined by the thermodynamics of energy conversion.

| **Electrode** | **Electrode reaction** | $V_{\mathbf{electrode}}$ **/ volt** |
|---|---|---|
| sodium | $Na \rightarrow Na^{+} + e^{-}$ | -2.72 |
| magnesium | $Mg \rightarrow Mg^{2+} + 2e^{-}$ | -1.55 |
| zinc | $Zn \rightarrow Zn^{2+} + 2e^{-}$ | -0.76 |
| hydrogen | $H_2 \rightarrow 2H^{+} + 2e^{-}$ | 0.00 |
| copper | $Cu^{2+} + 2e^{-} \rightarrow Cu$ | +0.34 |
| silver | $Ag^{+} + e^{-} \rightarrow Ag$ | +0.80 |

**Table 5.1 Standard electrode potentials**

## Electrode processes

Equilibrium can be reached if the metal electrode is held by an insulator so that there is no external current path (figure 5.4). The electrolyte in the solution is partially dissociated into positive metal ions and negative ions which both move through the solution in random thermal motion. Some of the metal ions strike the metal plate and some of those collect electrons from the metal, so they remain on the surface as neutral atoms. So the plate gains positive charge and the solution loses positive charge.

Metal atoms can also dissolve off the plate; they become ionised and diffuse into solution so that the plate gains negative charge and the solution gains positive charge. When the metal plate is first put into the electrolyte the rate at which positive ions flow from the metal into the solution is greater than the rate at which ions are deposited on the metal from the solution - because the concentration of ions is much greater in the metal than in the solution. So the solution gains excess positive charge, leaving behind on the metal plate an excess of negative charge. A charge double layer forms at the interface, and the resulting electric field across the boundary encourages the deposition of positive ions from the solution and discourages the metal atoms of the plate from going into the solution. An equilibrium is rapidly reached in which the plate is *negative* with respect to the solution, and the rate of deposition of metal ions is equal to the rate at which metal atoms dissolve from the plate. In this equilibrium the ions form and recombine at the same rate, and the mass of the plate does not change.

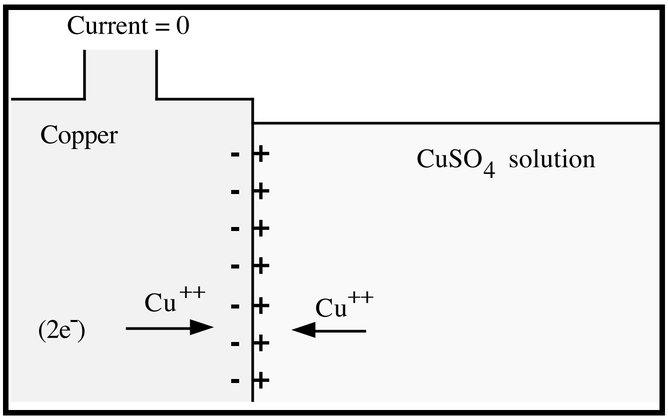


**Figure 5.4 Copper-copper sulfate electrode with no current**

When a copper plate is placed in copper sulfate solution, copper ions are simultaneously dissolved from and deposited on the metal plate:

$$Cu \rightleftharpoons Cu^{2+} + 2e^-$$

If the plate is *insulated,* a charge double layer rapidly builds up on the interface, thus equalising the rates of dissolving and deposition. The copper is in Nernst equilibrium.

## Currents through electrodes

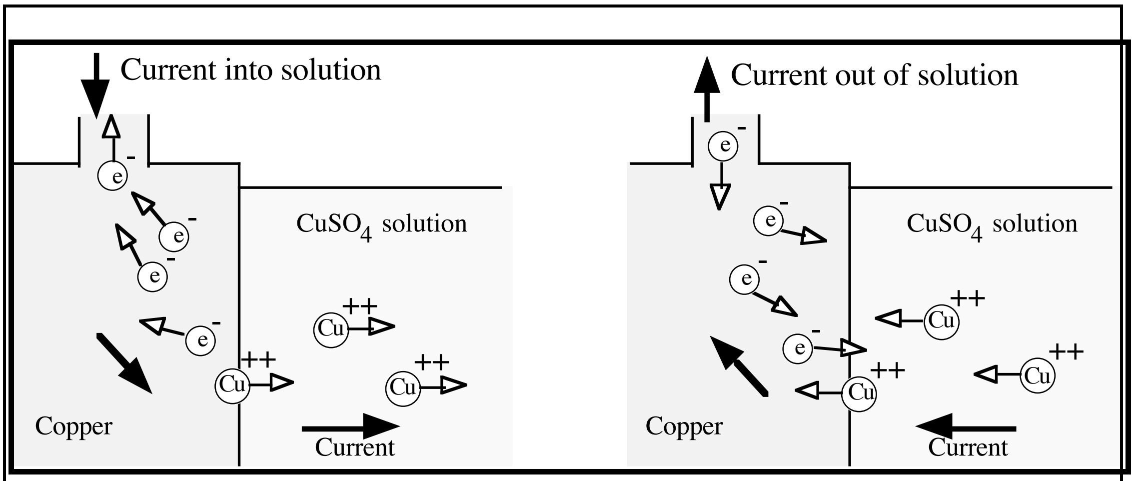


**Figure 5.5 Current in copper-copper sulfate electrode**

Current is carried by $Cu^{2+}$ ions in the solution and by electrons in the metal.

When current enters the metal plate from an external circuit, diffusion to and from the metal still leaves a net negative charge on the plate but the process is no longer self-limiting. Electrons flow out of the plate to the external circuit, and there is a corresponding flow of positive ions from the metal to the electrolyte. This flow of ions is no longer balanced by the deposition of ions from the solution; when charge flows into the metal plate, the metal dissolves. *The rate at which the metal dissolves is controlled by the current.*

Exactly the opposite happens when there is a current out of a metal electrode to an external circuit. Electrons flow into the plate and a corresponding amount of metal is deposited, again at a rate controlled by the current. In a situation like this, where the rate and direction of the reaction are totally determined by the current, we say the electrode process is electrochemically **reversible**.

Basically, a reversible electrode is one which can pass current without altering its chemical environment.

**Non-reversible electrodes**

To see what happens if the metal of the solid electrode and the metal ions in solution are different consider the example of a zinc electrode in copper sulfate solution. Zinc ions ($Zn^{2+}$) diffuse into the solution, and copper ions ($Cu^{2+}$) diffuse from the solution on to the zinc plate.

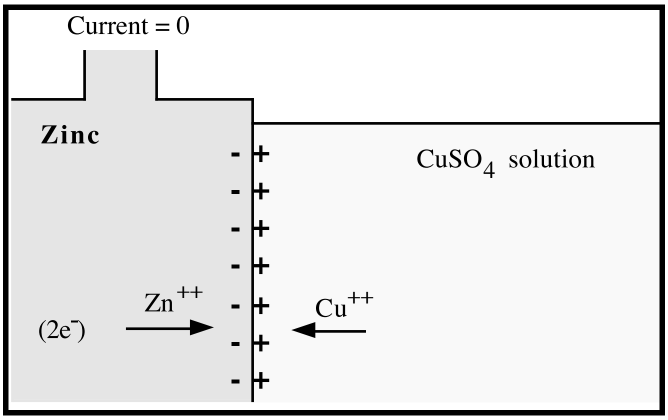


**Figure 5.6 Zinc-copper sulfate electrode - no current**

When a zinc plate is placed in $CuSO_4$ solution, zinc ions are dissolved and copper ions are simultaneously deposited on the plate.

$$Zn \rightarrow Zn^{2+} + 2e^-$$
$$Cu^{2+} + 2e^- \rightarrow Cu$$

If the plate is insulated, a charge double layer rapidly builds up, thus equalising the rates of these two reactions. There is no equilibrium, but a continuing transfer of zinc to the solution and copper to the plate.

A charge double layer builds up to give a steady state in which the copper ions are deposited at the same rate as the zinc ions dissolve. The plate gets coated with copper and the electrolyte becomes contaminated with zinc. The potential difference across the interface is such that there is no net current flow across the interface. However, unlike the case of the copper-copper sulfate electrode, there is a net flow of copper ions to the zinc plate, which is balanced by a net flow of zinc ions from the plate.

When there is current into the zinc plate from an external circuit the zinc dissolves even more rapidly (figure 5.7). A current into the zinc electrode corresponds to a flow of electrons *from* the electrode to the external circuit, and a flow of zinc ions from the zinc plate to the electrolyte. When the current is reversed the deposition of copper is speeded up, and dominates the flow of zinc ions into the electrolyte. *This process is NOT electrochemically reversible.*

**Reversible electrodes**

In the examples above the distinction between the reversible copper-copper sulfate electrode and the non-reversible zinc-copper sulfate electrode is clear: copper won't contaminate the copper sulfate, whereas zinc will. A considerable research effort has been expended in developing suitable electrodes for various purposes; electrodes for making biological measurements, electrical batteries etc. In many of these the property of electrochemical reversibility is an important consideration.

## 5-5 BIOELECTRODES

**Bioelectrodes** are specifically designed to make electrical contact with a living organism, e.g. for electrocardiographs, electroencephalographs, making electrical measurements on living cells, etc. Two vital characteristics are required of such electrodes.

- They must not irritate, or otherwise interact with the specimen.
- The electrode potential must be well-defined, so that it does not affect measurements.

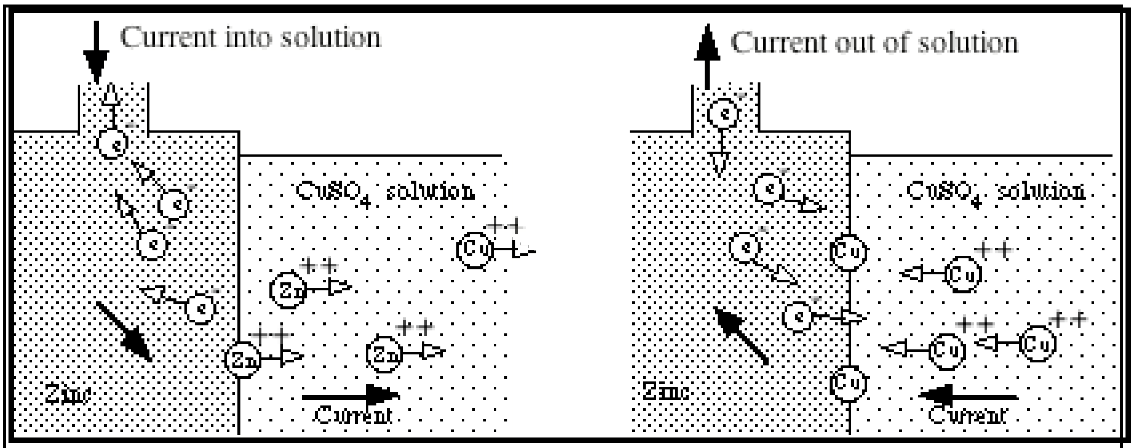


**Figure 5.7 Current through a zinc-copper sulfate electrode**
External current into the zinc electrode corresponds to a flow of electrons from the electrode, and a flow of zinc ions from the zinc plate to the electrolyte. The electrode is not electrochemically reversible. Current from the solution to the metal is carried by copper ions in the solution and by electrons in the metal. Copper is deposited on the zinc plate.

## Electrodes for electrocardiography

Early this century the usual electrocardiograph (ECG) electrode was a hypodermic needle, inserted under the skin. The electrode made contact with the patient, but it also hurt! Furthermore such electrodes satisfied neither of the above criteria. Nowadays contact is made to the patient's skin with specially designed electrodes. One very satisfactory design is a disposable stick-on electrode specially developed for electrocardiography, which is painless, non-irritating, reliable - and reversible! (See figure 5.8.)

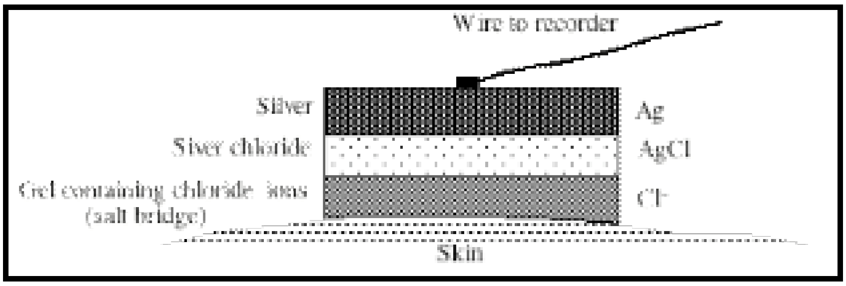


**Figure 5.8 Structure of a disposable ECG electrode (schematic)**
The silver chloride layer forms a reversible electrode with the chloride-containing gel. The gel makes a compatible bridge to human tissue.

The electrode consists of silver foil coated with silver chloride (AgCl). A proprietary gel containing chloride ($Cl^-$) ions provides an electrically conducting path between the silver chloride and the skin. A chloride electrolyte is desirable for this application as it is compatible with chloride ions in the body tissues and perspiration.

The actual electrode in the device - the contact between a solid and a solution - is the silver-silver chloride interface. The electrode reaction is:

$$Ag + Cl^- \rightleftharpoons AgCl + e^-$$

The electrode is a reversible **chloride electrode**. The interfaces between the silver chloride and the proprietary contact paste, and between the contact paste and the skin, have contact potentials resulting from the diffusion of ions. In practice the potentials across these interfaces are low and, more importantly, they change very little with changes of current during the measurement process. This is particularly important in operating theatres where stray currents passed through the electrodes by cardiac defibrillators must not prevent immediate observation of the electric signals from the heart.

### Calomel electrodes

Chloride is a very common ion in living organisms, so many bioelectrodes are based on it. One example is the **calomel electrode**. (Calomel electrodes were used to measure the potential across the membrane in the experiments on ion diffusion in video lecture E4, and in the cell potential measurements shown earlier in video lecture.)

This electrode consists of calomel (mercurous chloride, $Hg_2Cl_2$) in contact with mercury. The electrode reaction is

$$Hg_2Cl_2 + 2e^- \rightleftharpoons 2Hg + 2Cl^- .$$

This is another reversible electrode. It is linked to the organism to be investigated by a **salt bridge** containing a saturated solution of potassium chloride. In the electrodes used to study membrane potentials this salt bridge is contained in a bundle of fibres. In the electrodes used for measuring plant cell potentials it is contained within the very fine glass tube which is inserted into the cells. This bridge acts in the same way as the paste used to establish contact between the electrocardiograph electrode and the skin. The combination of a reversible electrode and a compatible salt bridge is often used as a **reference electrode**. Such reference electrodes are widely used in the life sciences.

## 5-6 BATTERIES

In the individual cells of batteries, electrode potentials are exploited to realise the conversion of chemical energy into electrical energy; each cell with its pair of electrodes is a **source of EMF**. The open-circuit cell potential, (which is equal to the EMF) is equal to the difference in the two electrode potentials. There are hundreds of chemically stable electrodes which can be used to make batteries..

The classic example is a relatively simple system known as the Daniell cell, one of the earliest practical batteries (figure 5.9). It usually consists of two half-cells, one copper-copper sulfate electrode and one zinc - zinc sulfate electrode, separated by a porous wall, usually an earthenware pot. It performs the same function as the salt bridge - keeping the different electrolytes substantially apart while still providing a conducting path for sulfate ions to diffuse through.

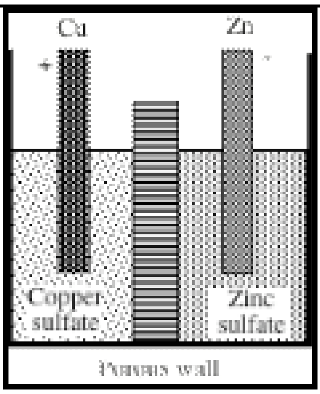


**Figure 5.9 A Daniell cell**

Net electrical energy is delivered by the pair of chemical reactions: zinc going into solution and releasing electrons and copper coming out of solution by picking up electrons. We can use the table of electrode potentials (table 5.1) to predict the cell voltage if both electrolytes are molar solutions (other concentrations will give different values).

Zinc electrode: $Zn \rightarrow Zn^{2+} + 2e^-$ ; $V_{Zn} = -0.76$ V.

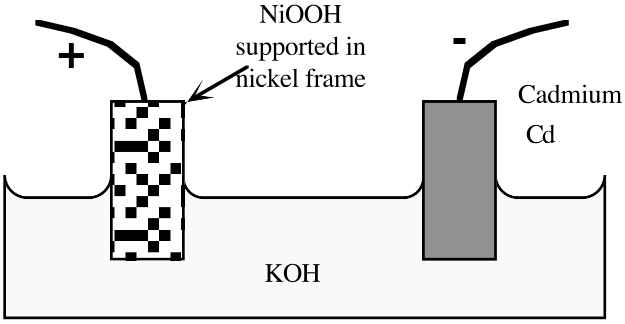
Copper electrode: $Cu^{2+} + 2e^- \rightarrow Cu$ ; $V_{Cu} = +0.34$ V.

Total: $Zn + Cu^{2+} \rightarrow Zn^{2+} + Cu$ ; $V_{cell} = V_{Cu} - V_{Zn} = 1.10$ V.

This calculation shows that the copper is positive with respect to the zinc. There is no way of measuring the potential in the electrolytes.

## The Nicad battery

The nickel-cadmium battery (often called a nicad battery) is a fully rechargeable cell frequently used in cordless power tools, calculators, and portable electronic instruments. The positive electrode of a fully charged nicad cell consists of nickel oxide hydroxide supported on a nickel frame, while the negative electrode is cadmium metal. Both these electrodes are reversible in the potassium hydroxide electrolyte.



**Negative electrode**

The reaction at the negative electrode is the simpler. When no current is drawn a Nernst equilibrium exists, the reaction being self-limited by the build-up of negative charge on the electrode.

$$\mathrm{Cd} + 2\mathrm{OH}^- \rightleftharpoons \mathrm{Cd\,(OH)_2}^{2-} .$$

When the cell is supplying current to an external circuit, electrons flow from this electrode, and the reaction proceeds to form cadmium hydroxide - the cadmium is *oxidised* to cadmium hydroxide.

$$\mathrm{Cd} + 2\mathrm{OH}^- \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftharpoons}} \mathrm{Cd(OH)_2} + 2e^- .$$

When the cell is being charged this reaction is reversed. Electrons are supplied *to* the cadmium electrode, and the hydroxide is reduced back to metallic cadmium.

**Positive electrode**

The positive electrode is a hydrated oxide of nickel. During discharge it is reduced to divalent nickel hydroxide.

When no current is drawn a Nernst equilibrium exists, positive charge building up on the electrode.

$$\mathrm{NiOOH} + \mathrm{H_2O} \rightleftharpoons \mathrm{Ni(OH)_2}^{+} + \mathrm{OH}^- .$$

When the cell is supplying current to an external circuit, electrons flow into the anode from the external circuit and the reaction proceeds to form nickel hydroxide.

$$\mathrm{NiOOH} + \mathrm{H_2O} + e^- \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftharpoons}} \mathrm{Ni(OH)_2} + \mathrm{OH}^- .$$

Thus, during the discharge of this cell cadmium is oxidised to its hydroxide whereas the hydrated nickel oxide is reduced to nickel hydroxide. The potassium hydroxide electrolyte is unchanged during the cell reaction; its purpose is to provide plenty of $\mathrm{OH}^-$ ions which are exchanged between the electrodes, but not used up.

Both electrodes of a nicad cell are electrochemically reversible in the same electrolyte - a property which simplifies the construction of practical nicad cells.

## Concentration cells

We have seen that electrode potentials, as described by equation 5.1, can be explained as having two parts - one contribution from a standard electrode potential and another arising from the concentration of the electrolyte . That model suggests that we ought to be able to get a net cell voltage by using two electrodes with the same metal and electrolyte and a salt bridge (or a porous pot) with different electrolyte concentrations in the two compartments. Such a device, which is called a concentration cell, works but the voltage is often small.

### Example

A concentration cell consists of two silver-silver nitrate electrodes with 1.0 molar silver nitrate in one compartment and 0.10 molar on the other side. When we write down the difference between the electrode potentials, the standard electrode potentials cancel out and we are left with the difference in concentration potentials, from equation 5.1:

$$\begin{aligned} V_{cell} &= \frac{kT}{ze}\ln\left(\frac{C_1}{C_{ref}}\right) - \frac{kT}{ze}\ln\left(\frac{C_2}{C_{ref}}\right) \\ &= \frac{kT}{ze}\ln\left(\frac{C_1}{C_2}\right) \ . \end{aligned}$$

Supposing that the temperature is 20°C or 293 K and given that $z = 1$ for the silver ion, you might remember that the factor in front of the log function is about 25 mV. Putting in the ratio of concentrations gives a potential of 25 mV × ln(10) which is equal to 58 mV or about 0.06 V - not much of a battery but you could improve it by increasing the concentration ratio.

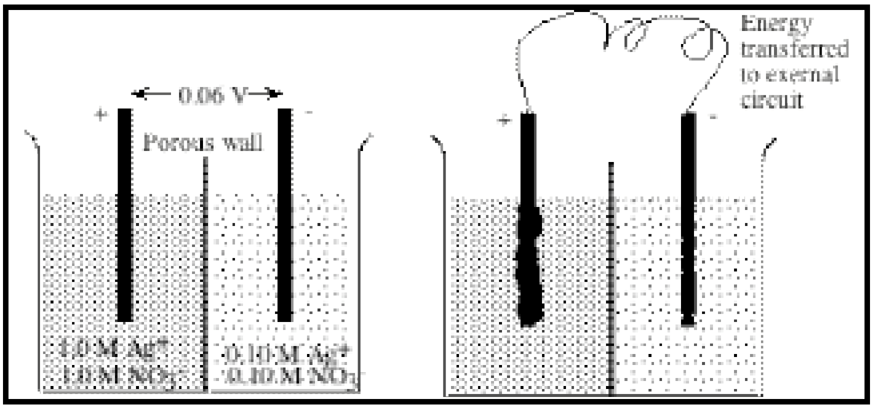


Figure 5.10 A concentration cell

Remember that this result is the potential difference between the electrodes when they are not connected to an external circuit - the cell just sits at a potential difference of 58 mV and there is a chemical equilibrium. When you draw energy from the cell by completing the circuit the chemical reactions start up to supply the energy, and the potential difference between the terminals will drop. The reactions proceed in a direction that will try to equalise the concentrations of silver ions in the two compartments That can be achieved by moving electrons along the external connecting wire from the low concentration electrode to the high concentration side, so that silver ions can pick up electrons and come out of solution as silver atoms. An equal amount of silver goes into solution on the low concentration side.

## Summary of principles: electrodes and cells

- Electrode potential = standard electrode potential + concentration potential.
- Standard electrode potential = cell potential with molar electrolytes and hydrogen reference electrode.
- Cell potential = difference in electrode potentials.

## 5-7 ELECTROLYSIS

**Electrolysis** is the process in which electric currents from an external source supply the energy necessary to make a chemical reaction take place. It is just the opposite of what occurs in a battery, where the energy of a chemical reaction is converted into electrical energy. Although electrolysis occurs in charging and discharging a battery, the term electrolysis is usually used only when the primary interest is in the chemical products of the process.

Consider two copper electrodes in a copper sulfate solution. The reaction caused by a current is simply the transfer of copper from one electrode to the other. This very simple system - two electrodes of the same metal in an electrolyte containing that metal's ion - illustrates the principles of electrolysis, and permits us to investigate further the subtleties of electrode processes.

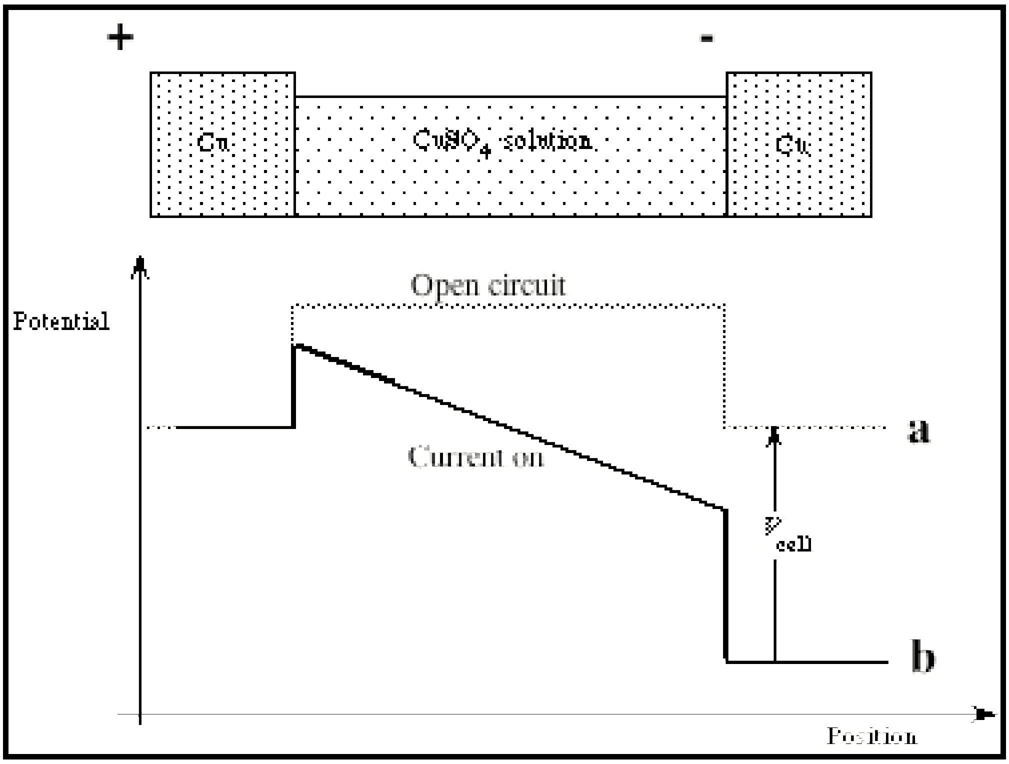


**Figure 5.11 Variation of potential through an electrolytic cell**

(a) No current (light line)

(b) Current causes transfer of copper between electrodes (heavy line)

First, take the case where there is no current. Each electrode is in Nernst equilibrium with the electrolyte. Diffusion of ions from each plate establishes a negative charge on the plate. The resulting charge double layer limits the rate of diffusion of ions from the plate so that it is equal to the rate of diffusion of ions from the electrolyte to the plate.

If we arbitrarily take the potential of the copper sulfate electrolyte as zero, then each copper electrode will be at some negative potential relative to that, i.e. at the appropriate Nernst equilibrium value for the particular electrolyte concentration and temperature. The observable quantity - i.e. the measured cell voltage - is the difference of these two equal Nernst potentials, i.e. 0 V.

When a source of EMF is connected to the cell, one plate is made positive with respect to the other, so there is a current through the cell, as $Cu^{2+}$ ions. As a result copper is removed from the positive plate, and deposited on the negative plate.

By maintaining this potential difference across the cell we have considerably changed the potential distribution in the cell. This applied voltage, $V_{cell}$, is equal to the sum of three components. (See figure 5.11.).

(1) There is an interface potential at the positive electrode which is less than the Nernst value. The rate at which copper dissolves is greater than the rate at which it is deposited on the electrode. so there is a net flow of copper into the electrolyte.

(2) At the negative electrode the interface potential now exceeds the Nernst value, and there is a net flow of $Cu^{2+}$ ions from the solution to the electrode, where a deposit of copper is built up.

(3) There is a potential gradient across the electrolyte and an associated electric field within the electrolyte which drives the flow of $Cu^{2+}$ ions between the electrodes.

The difference between the Nernst equilibrium potentials and the actual potential difference at the electrode when there is an electric current is called the **overpotential** of the electrode. These overpotentials and the resistive potential drop across the cell must be established to make the reaction proceed. However they represent wasted energy. They can be minimised by suitable cell design and a low electrolysis rate, but some potential difference is required, and hence some energy must be dissipated to make the process proceed at a finite speed.

## Applications of electrolysis

### *Electroplating*

Electrolysis may be used to deposit a thin but tough layer of a selected metal on to a base of another metal, either for protection or decorative purposes. Examples include the following.

- Cutlery is plated with silver to give an attractive finish.
- Steel parts are cadmium plated to prevent corrosion.
- Steel car fittings are chromium plated both to prevent rust and provide an attractive finish. To make a tough adherent coating the fitting is plated with nickel before the chromium plating.

In electroplating, the object to be coated is the negative electrode of an electrolytic cell, and the positive electrode and electrolyte are chosen to deposit the desired material.

The video lecture shows plumbing fittings being nickel plated. The positive electrode is a basket of nickel pellets, the fittings, suspended in the plating bath from conducting rods, form the negative electrode. The electrolyte is a mixture of nickel salts.

### *Electropurification of metals*

Electrolysis is also used for the purification of certain metals. Of particular importance is the electropurification of copper. The impure metal is the positive electrode of an electrolytic cell, and pure metal is deposited on the negative electrode. Impurities either dissolve in the electrolyte, or fall as sludge to the bottom of the tank. The cell potential is adjusted so that copper is deposited out in preference to other metallic ions.

## Mass transfers in electrolysis

In §3-4 of chapter E3 we discussed the rates of mass transfer associated with electric currents. The principles described there can be used to calculate the chemical amounts and masses of ions transported during electrochemical processes.

## CONCLUSION

In this chapter we have discussed the fundamental processes occurring at the interface between an electrode and an electrolyte, including the idea of a chemical reaction controlled reversibly by an electric current and the related idea of a Nernst equilibrium at a chemically reversible electrode. We have shown how these ideas apply to the design of bioelectrodes, to the nicad cell, and to electrolysis. The same ideas are equally applicable to other aspects of electrochemical science and engineering - electrosynthesis, fuel cells, electroformation of structures, desalination, corrosion, electrophoresis, and so on.

## REFERENCE

For more information try chapter 17 of *Chemistry*, by Steven S. Zumdahl, Lexington, Mass: D.C. Heath & Co. 1989.

# POST-LECTURE

## 5-8 QUESTIONS

**Q5.1** It is possible to 'make contact' with a plant, animal or a solution by 'just sticking in a bit of wire'. Why is this simple approach unsatisfactory in many experimental or clinical situations. give brief descriptions and explanations of some electrodes used in chemistry, botany, physiology and medicine.

**Q5.2** The electrode potential of cadmium relative to a standard hydrogen reference electrode is -43 mV in a particular electrolyte, and the electrode potential of gold relative to the same reference electrode in the same electrolyte is 80 mV. What will be the cell potential when these metals form a cell in this electrolyte?

**Q5.3** When electrodes of zinc and palladium are placed in an electrolytic cell the palladium electrode is 1.59 V *positive* with respect to the zinc. The electrode potential of zinc relative to a standard hydrogen reference electrode is -0.76 V. What is the electrode potential of palladium relative to this standard electrode?

**Q5.4** In questions 5.2 and 5.3 electrode potentials are specified as potentials *relative to a standard hydrogen reference electrode*. Why is the electrode potential specified in this manner rather than absolute values being given?

**Q5.5** The electrode potential, relative to a standard hydrogen reference electrode, of copper in a solution containing copper ions of concentration.0.10 mol.L$^{-1}$, is 0.31 V. This measurement was made at room temperature (297 K).

What will the electrode potential of the copper be if the concentration of the solution is increased by a factor of 10, the cell temperature remaining unchanged?

**Q5.6** A student, in replying to an examination question, writes:

> *'Corrosion of metal is due to electrical currents of electrochemical origin. Stop that current, and you stop corrosion.'*

Comment on the correctness or otherwise of this statement on the basis of your knowledge of electrochemistry.

**Q5.7** When a silver-zinc cell delivers electrical energy to a load, silver oxide ($Ag_2O$) is reduced to silver at one electrode, and zinc is oxidised to zinc oxide (ZnO) at the other. Both electrodes are in a potassium hydroxide (KOH) electrolyte.

Write down the chemical equations for the reactions which occur at each electrode.

Which electrode is the positive electrode of this battery? Does the strength of the hydroxide electrolyte change as a result of these reactions?

(The silver-zinc cell is an efficient reversible cell commonly used in spacecraft.)

**Q5.8** a) What properties are desirable for an electrode designed for biological applications?

b) Describe the structure and properties of a stick-on ECG electrode. In your reply discuss how this electrode satisfies the requirements you listed in your answer to part (a) of this question.

**Q5.9** An electrolytic cell consists of a copper electrode in an electrolyte containing $Cu^{2+}$ ions, separated by a porous partition from a silver electrode in an electrolyte containing only $Ag^+$ and negative ions.

When there is a constant current through this cell for a specific time, 0.010 kg of copper is dissolved off the copper electrode.

a) Will the mass of the silver electrode *increase* or *decrease* as a consequence of the passage of this current?

b) By how much will the mass of this electrode change?

Molar masses: Ag: 108 g.mol$^{-1}$; Cu: 63.5 g.mol$^{-1}$.

Hint: assume that the negative ions present do not take part in the interactions at either electrode.

# E6 MAGNETISM: FIELDS AND FORCES

## OBJECTIVES

### Aims

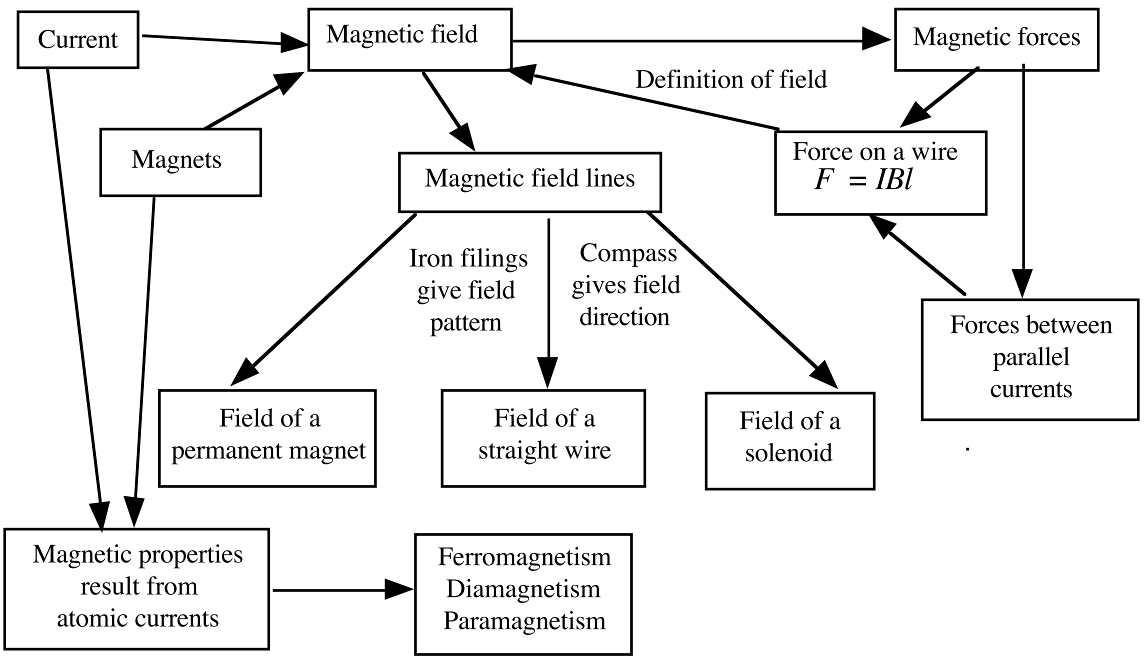
In studying this chapter you should aim to understand the nature of magnetism and the concepts of magnetic field and magnetic forces. You should be able to display that understanding by describing and discussing examples of magnetic effects. You should learn how to calculate magnetic fields due to currents in straight wires and magnetic forces on current elements and moving particles.

### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms:
   *magnet, magnetic field, magnetic field line, magnetic field strength, tesla, north pole [north-seeking pole], south pole [south-seeking pole], solenoid, permeability of free space, domain, ferromagnetism, diamagnetism, paramagnetism.*
2. Describe examples of magnetic effects in everyday life and in the life sciences.
3. Describe the behaviour of a compass needle in a magnetic field.
4. Discuss the form and origin of the Earth's magnetic field.
5. Sketch the magnetic field lines in the vicinity of a bar magnet, a solenoid, pairs of bar magnets placed end to end and a very long straight current-carrying wire.
6. Calculate the magnitude and direction of the magnetic force on
   (a) a straight current-carrying conductor in a uniform magnetic field,
   (b) a charged particle moving through a magnetic field.
7. State the order of magnitude of typical magnetic field strengths.
8. Describe the operating principles a DC motor, a moving coil meter, a loudspeaker and the magnetic deflection of the electron beam in a TV picture tube.
9. Calculate the magnitude and direction of the magnetic field near a long, straight current-carrying conductor.
11. Describe the interaction of parallel current-carrying conductors.
12. Describe and explain the magnetic properties of ferromagnetic, diamagnetic and paramagnetic materials.

### CONCEPT DIAGRAM



# LECTURE

## 6-1 WHY STUDY MAGNETISM?

Magnetism is familiar to most people in terms of the attraction of iron filings to a permanent magnet and the alignment of a compass in the Earth's magnetic field. Magnetic phenomena are also the basis of much of our modern technology. Visual and audio information is stored on magnetic tapes, and much of the information used with computers is stored magnetically. Electric motors are driven by the magnetic force on an electric current, as are loudspeakers. The alignment of atomic nuclei in a magnetic field is the basis of nuclear magnetic resonance spectroscopy, an analytical technique of increasing importance in chemistry, biology and medicine. Beyond the confines of our planet magnetism is important in many astrophysical phenomena including pulsars - massive, compact, rotating stars with intense magnetic fields. More speculatively, it is suggested that the Earth's magnetic field is used to some extent as a navigational aid by migratory birds and other animals.

In this chapter we discuss the nature of magnetic fields, the force on a current-carrying conductor in magnetic fields, and the magnetic properties of matter.

## 6-2 MAGNETIC FIELDS

When iron filings are sprinkled around a magnet they do not fall randomly. In any small region they tend to point in the same direction, and often line up head to tail forming a chain of filings. The orientation of the filings in any small region is parallel to the direction of the magnetic field in that region. Thus the pattern of iron filings in the vicinity of a bar magnet shows the pattern of the magnetic field near the magnet - they help us to visualise the magnetic field.

### Magnetic field lines

A family of lines can be drawn to indicate the orientation taken up by iron filings in the vicinity of a magnet. When arrows are added to show the actual direction of the magnetic field the lines are called **magnetic field lines**.

When they are correctly drawn, the areal density of the field lines (i.e. the number of lines per area normal to the lines) is proportional to the **magnetic field strength**. Thus magnetic field lines represent the direction and strength of a magnetic field just as electric field lines represent the direction and strength of an electric field.

The magnetic field lines near a bar magnet are shown in figure 6.1.

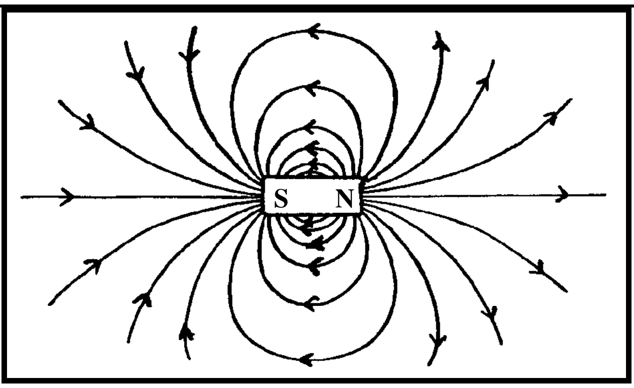


**Figure 6.1 Magnetic field of a bar magnet**

### The direction of a magnetic field

When a small bar magnet is freely suspended in the Earth's magnetic field it rotates until one end points north. This end is called the **north-seeking pole** (frequently abbreviated to **north pole**) of the magnet; the other end is the **south-seeking pole**.

When such a magnetic compass is placed in the magnetic field of a bar magnet it aligns itself in the direction of the local magnetic field. It is oriented so that its north-seeking pole always points along the field line which runs towards the south-seeking end of the magnet. By convention we put the arrows on the field lines, in the direction in which the compass points, so that the field lines of the magnet are directed from its north-seeking pole to its south-seeking pole as shown in figure 6.1.

### The sense of the Earth's magnetic field

When a small compass needle is allowed to rotate freely in the Earth's magnetic field it will settle down to point roughly towards the North geographic pole of the Earth. Thus the magnetic field lines of the Earth run from the southern hemisphere towards the northern hemisphere.

The external magnetic field of the Earth may be represented by assuming that the Earth's core is magnetised with its north-seeking pole towards the geographic South, as shown in figure 6.2.

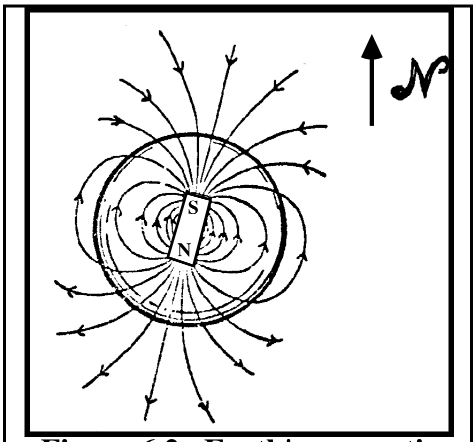


**Figure 6.2. Earth's magnetic field**

**Demonstration: Forces between permanent magnets**

The forces between permanent magnets are investigated by placing a vertical bar magnet in a puck which floats over the surface of an air table, and bringing a hand-held magnet towards this floating magnet. This experiment shows that **like poles repel** each other and **unlike poles attract** each other.

### Magnetic fields of bar magnets

Examples of magnetic field patterns for a pair of identical bar magnets are shown in figure 6.3. Note that in the case of unlike adjacent poles, field lines start from the north-seeking pole of one magnet and terminate on the adjacent south-seeking pole of the other. For adjacent like poles the field lines at off-axis points between the magnets appear to be squeezed together because the magnitude of the total field is greater than that of a single magnet. On the other hand, at points along the common axis of the magnets the total field is zero because the individual fields have opposite directions.

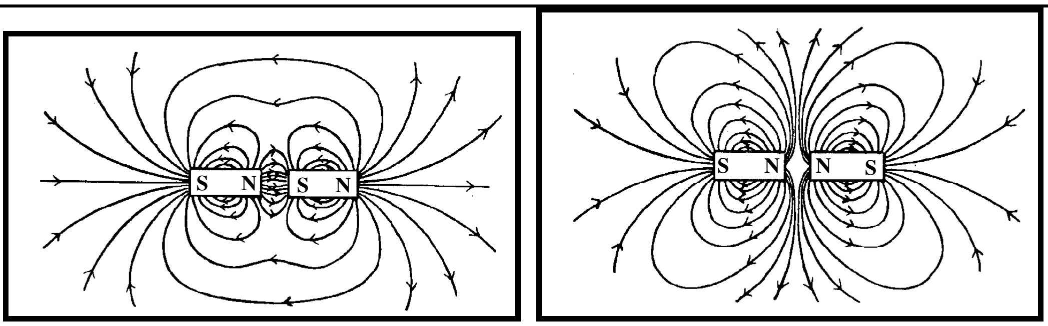


**Figure 6.3 Field patterns for pairs of bar magnets**

## 6-3 FORCE ON A CURRENT IN A MAGNETIC FIELD

A current-carrying conductor in a magnetic field has a **magnetic force** exerted on it. Since the force depends on the length (*l*) and orientation of the wire, it is simplest to consider the force on the current in a short straight section of the wire (called a current element).

The direction of the force (***F***) is perpendicular to both the direction of the current and the magnetic field as shown in figure 6.4. Since there are two directions perpendicular to the plane containing the directions of the field and the current (the shaded plane in figure 6.4) we need a rule to define one of them: if you curl the fingers of your right hand form the direction of the current towards the field direction, then your thumb gives the direction of the force. (There are other mnemonics for this direction, for example arrange the thumb and the next fingers of your left hand roughly at right angles. They represent force, field, current in the order thumb, first finger, second finger.)

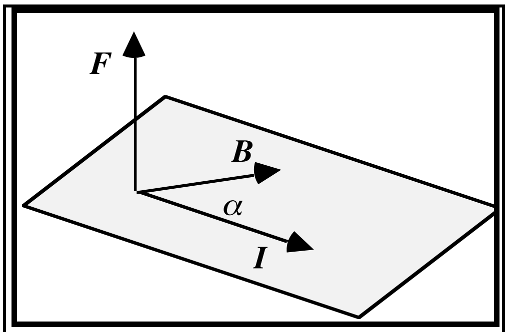


**Figure 6.4 Direction of the force on a current element**
The force is perpendicular to the plane defined by the directions of the current and the field.

The magnitude of the force is

$$F = IBl\sin\alpha \qquad .... (6.1)$$

Here $I$ is the current, $B$ is the magnitude of the magnetic field, $l$ is the length of the segment of wire and $\alpha$ is the angle between the directions of the current and magnetic field. Note that this relation applies only to a short current element; to find the force on a complete wire or a more complex system, it is necessary, in principle, to combine the forces acting on all the parts of the system.

**Demonstration**

The magnetic force is investigated by locating a wire carrying a current of several amperes in the strong field in the gap of a magnetron magnet. Springs at the end of the wire constrain its motion so that its vertical displacement is proportional to the magnetic deflecting force. (See figure 6.5.)

A series of measurements with this apparatus verifies the relationship for the magnitude $F$ of the magnetic force when a length $l$ of the wire, carrying current $I$, is placed perpendicular to a uniform magnetic field of magnitude $B$

$$F \propto IB\bullet$$

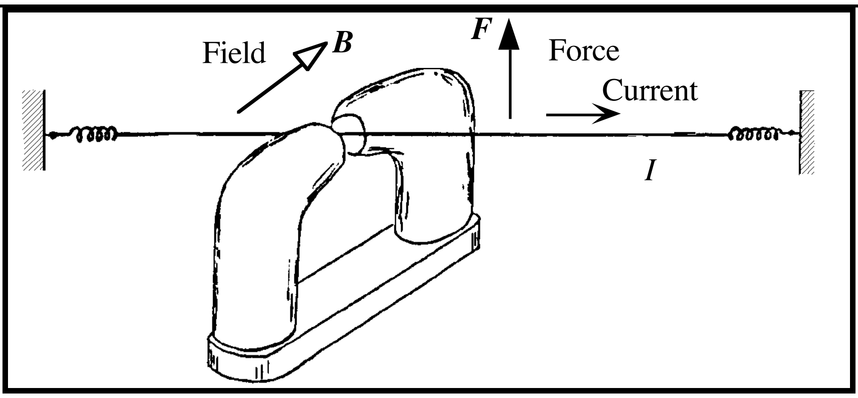


**Figure 6.5 Apparatus for investigating the force on a current in a magnetic field**

- The magnitude of the magnetic force is proportional to the current.
- The force is perpendicular to the field.
- The force is also perpendicular to the current.
- The direction of the force reverses when current direction is reversed.
- The direction of the force is reversed when the magnetic field's direction is reversed (but remains unchanged when the direction of both the current and the magnetic field are reversed).

Since this interaction between current and magnetic field is used to define the magnitude of the field the constant of proportionality in the relation above is arbitrarily set exactly equal to the number one. So the relation becomes

$$F = IBl.$$

Remember that this formula applies only for the special case of a current element at right angles to a uniform magnetic field ($\sin\alpha = 1$ this case).

From equation 6.1 we can see that the SI unit of magnetic field strength is the newton per ampere per metre ($\mathrm{N.A.^{-1}.m^{-1}}$) which is given the special name tesla (symbol T).

## Some typical magnetic fields

***Magnetron magnet.*** This powerful permanent magnet can be imagined as a bar magnet bent around to give a narrow gap between the poles, and consequently a strong magnetic field within this gap. For a magnetron magnet, $B \approx 0.5$ T.

***NMR spectrometer.*** Very strong magnetic fields are normally produced by superconducting magnets cooled to liquid helium temperature. For the superconducting magnet of the nuclear magnetic resonance (NMR) spectrometer shown in the TV program $B \approx 10$ T.

***Earth's magnetic field*** is around $10^{-4}$ T at the surface of the Earth.

***Interstellar space.*** There is a magnetic field in interstellar space, albeit a small one. Here $B \approx 10^{-9}$ T.

***Pulsar***. By contrast the magnetic field of pulsars - compact, dense, spinning neutron stars - is enormous: $B \approx 10^{8}$ T.

## 6-4 APPLICATIONS OF MAGNETIC FORCE

### Electric motors

A simple DC electric motor consists of a coil of wire pivoted in a magnetic field. A current flows down one side of the coil, and up the other side. As a consequence, equal and opposite forces act on the coil. The resulting torque will cause the coil to rotate (see chapter FE3).

By arranging for the direction of the current to reverse every time the coil turns through 180° the direction of this torque will not change as the coil rotates. The coil will continue to turn in the same direction.

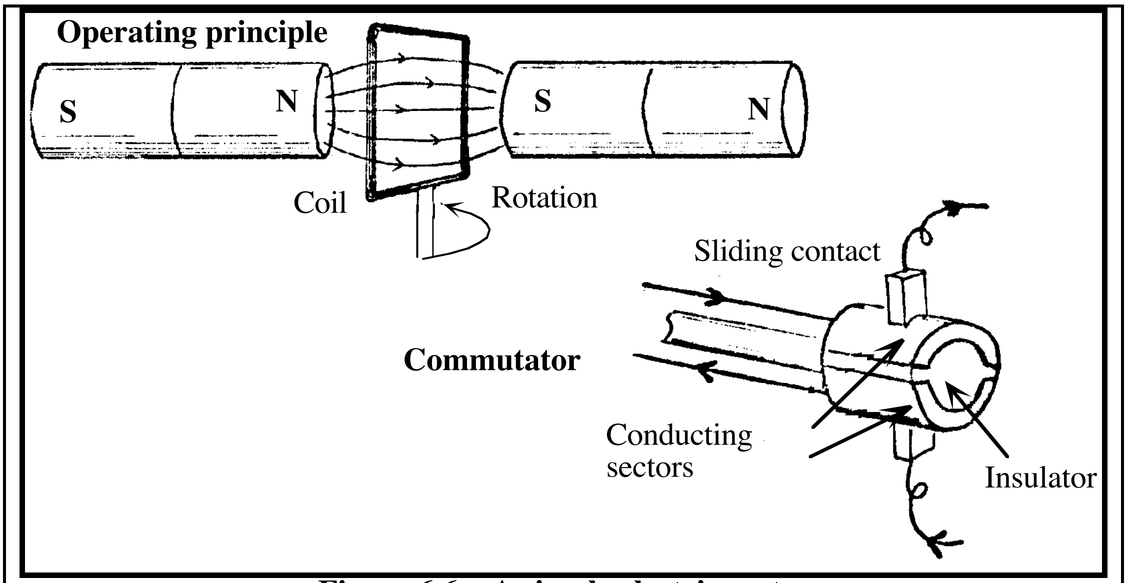


**Figure 6.6 A simple electric motor**

This current reversal is achieved by feeding the current into the coil through a commutator (see figure 6.6b). This device consists of an insulating disk with two conducting sectors mounted on its circumference. These are attached to each end of the coil, and are alternately connected to the positive and negative terminals of a current source as the coil rotates.

Modern industrial motors are much more complex than this. There are frequently several sets of coils to provide a more uniform torque, and the coils are wound on an iron armature to increase the efficiency. Most are especially designed to work on alternating current (AC) rather than direct current (DC). There are many variations and refinements of the simple motor shown in the TV program. In all cases they operate as the consequence of a force on a current in a magnetic field.

**Electric meters**

A moving-coil meter measures an electric current by passing it through a coil suspended in a magnetic field. Its principle of operation is similar to that of a DC electric motor. Here a circular iron cylinder is located inside the coil and the pole pieces of the magnet are shaped to provide a radial magnetic field with uniform magnitude. A spring is attached to the coil, which rotates until the magnetic torque on the coil is balanced by the restoring torque of the spring.

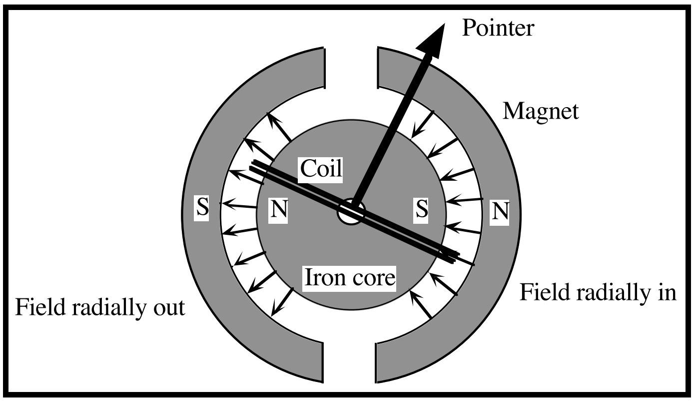


**Figure 6.7 Moving coil current meter**

**Loudspeakers**

In loudspeakers a circular coil is placed in the gap between the poles of a specially shaped permanent magnet, so that the field is radial. The current goes in a circular path in the gap between the poles of the magnet (see figure 6.8). The interaction of this current with the radial field gives an axial force on this coil, which drives the loudspeaker cone.

Notice the difference in the shapes of the field patterns for the meter (figure 6.7) and the loudspeaker (figure 6.8). In the meter, the field is arranged so that the total force on the coil is zero while the torque of the magnetic forces is not zero; so the coil rotates. In the case of the loudspeaker, there is no net torque but the total force drives the coil in an oscillating motion.

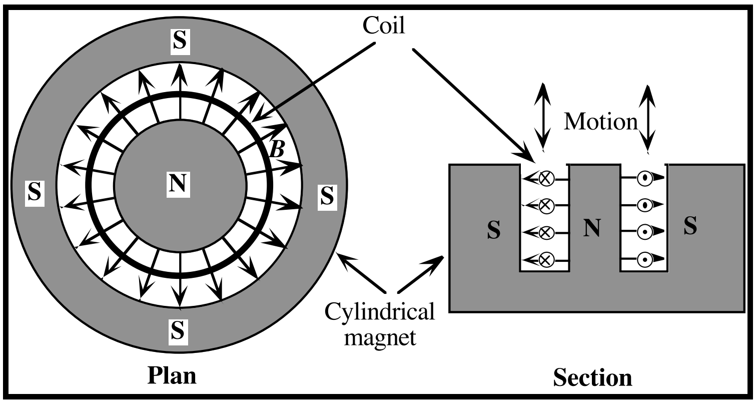


**Figure 6.8 Principle of operation of a loudspeaker driver**

The magnetic field is radially outwards everywhere in the gap between the magnet poles

## 6-5 MAGNETIC DEFLECTION OF ELECTRON BEAMS

A beam of charged particles flowing through a vacuum constitutes an electric current. Hence such a beam can be deflected by a magnetic field. The direction of the force is perpendicular to both the magnetic field and the particle's velocity so the beam is pulled sideways. The magnitude of the magnetic force on a particle with charge $q$ moving at speed $v$ at and angle $\alpha$ to the magnetic field $\boldsymbol{B}$ is

$$F = qvB\sin\alpha\ . \qquad \text{... (6.2)}$$

The electron beam which strikes the screen of a TV picture tube to produce an image is scanned across the screen using magnetic forces. The magnetic field is produced by passing a variable current through coils in the tube.

Magnetic deflection forces are also the basis of the operation of scientific instruments such as the mass spectrometer and the cyclotron.

## 6-6 WHAT CAUSES MAGNETIC FIELDS?

### Electricity and magnetism - a historical perspective

As separate phenomena, electricity and magnetism have been known for thousands of years. By the early 19th Century Volta's invention of the battery had made substantial electric currents available to experimenters. A remarkable connection between electricity and magnetism then became apparent. Hans Christian Oersted was probably the first person to observe the deflection of a magnetic compass by an electric current - an observation which is repeated in the TV program.

### Magnetic field of a solenoid

The effect produced by the current flowing through a straight wire is rather small - not much greater than that produced by the Earth's magnetic field. To intensify this effect the wire is formed into a cylindrical coil of many turns - a **solenoid**. A current through such a coil produces an iron filing pattern similar to that of a bar magnet.

Inside a long solenoid, away from the ends, the field is particularly uniform, with the field lines being straight and parallel to the axis of the coil.

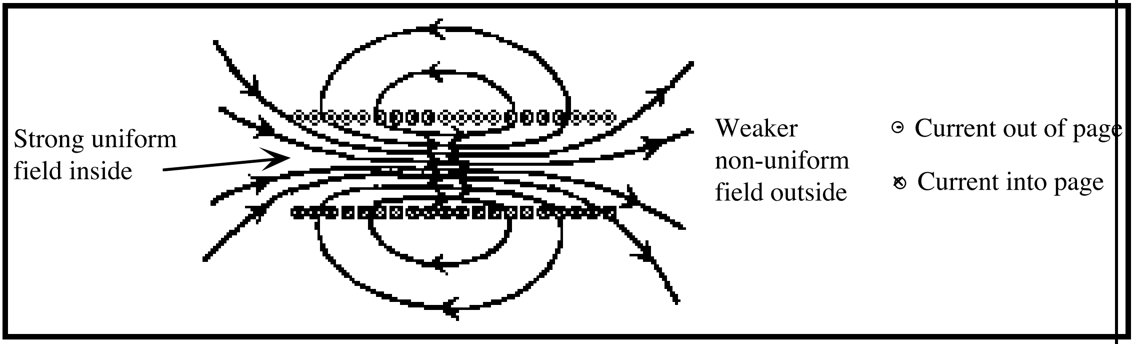


**Figure 6.9 Magnetic field of a solenoid**

**Demonstrations**

- Both iron filings and a small compass show that the magnetic field lines around a straight wire are circles concentric with the wire. The direction of this field reverses when the current is reversed.
- The strength of this field is measured with a **magnetometer**. The magnetometer used in the TV program is an electronic device which generates a voltage proportional to the strength of the magnetic field at the tip of a movable probe (strictly of the component of the magnetic field parallel to the axis of this probe).
- The magnetometer and a chart recorder are used to demonstrate that the magnitude of the magnetic field is proportional to the current and that the field drops off as the distance of the measurement point from the wire increases as described by equation 6.3.

- The direction of the magnetic field reverses when the direction of the current in the wire is reversed.

**Magnetic field of a long straight conductor**

The magnetic field lines in the vicinity of a long straight wire carrying a current form concentric circles around the wire (figure 6.10) and the direction of the field at any point is tangential to a field line.

In air or vacuum the magnitude of the field at a point is determined by the current $I$ in the wire and the distance $r$ of the point from the wire:

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{... (6.3)}$$

where the constant $\mu_0$ is called the **permeability of free space**. Its value is exactly $4\pi \times 10^{-7}$ Wb.A.m$^{-1}$.

The directions of the current and the field for a straight wire can be remembered by using the *right hand grip rule* (figure 6.11): if the wire is clasped by the right hand so that the thumb points in the direction of current flow, the fingers will curl around the wire in the sense of the magnetic field.

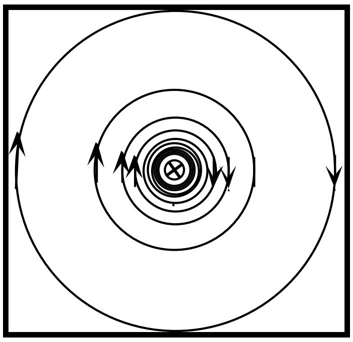


Figure 6.10 Magnetic field around a long straight wire

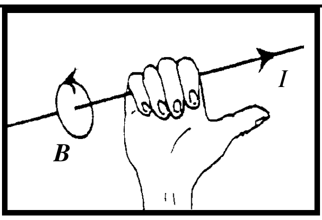
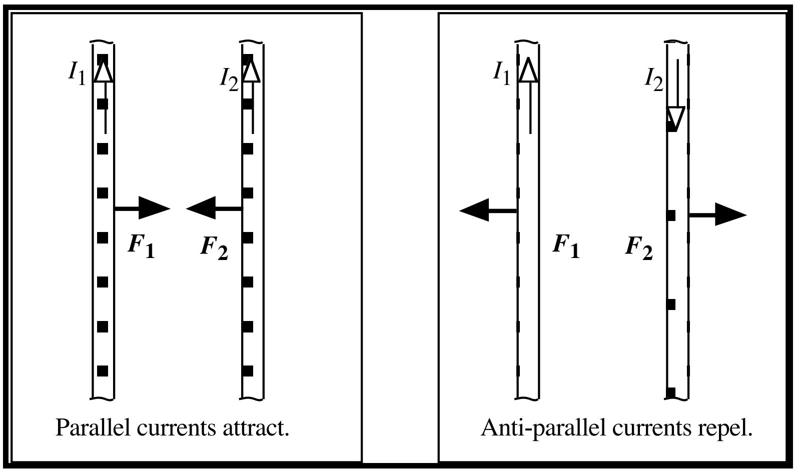


Figure 6.11 Right hand rule

## 6-7 FORCES BETWEEN CURRENT-CARRYING CONDUCTORS

Electric currents interact with each other through their magnetic fields. Consider two parallel, current-carrying conductors (figure 6.12).



Parallel currents attract.

Anti-parallel currents repel.

Figure 6.12 Forces between parallel wires

The current through the right-hand conductor produces a magnetic field. The interaction of the current in the left hand conductor with this field gives rise to a force on the left-hand conductor. Similarly the current through the left-hand conductor also produces a magnetic field. The interaction of the current through the right-hand conductor with this field gives rise to an *equal and opposite* force on the right-hand conductor. This can be summarised as follows.

**Parallel currents attract. Opposed currents repel.**

Interaction between the current in different parts of a single conductor can cause that conductor to collapse (the pinch effect). In the TV program we show a hollow copper conductor - part of a lightning arrester - which was crushed by the enormous current which flowed through it when struck by lightning. Such forces are significant in understanding the properties of various electrical discharge phenomena such as circuit breaker arcs, and the very high temperature ionised gases (plasmas) of interest in controlled thermonuclear fusion research.

## 6-8 MAGNETIC MATERIALS

Magnetic fields exist

- in the vicinity of certain metallic materials, when these are specially treated (e.g. bar magnets and magnetron magnets),
- when an electric current flows through a wire.

In particular, the magnetic field distribution revealed by patterns of iron filings near a bar magnet and a solenoid of similar shape are very much alike. These two phenomena are closely related.

The magnetic properties of magnets result from circulating electric currents at an atomic level. Around every atom is a moving cloud of electrons. Each orbiting electron constitutes a small electric current which produces a magnetic field.

In addition to its orbital motion, an electron has an intrinsic spin and an associated magnetic field. Furthermore, the nucleus of an atom also has a spin and an associated magnetic field. It is the magnetism of the individual atoms and nuclei and their interactions which determines the magnetic properties of materials.

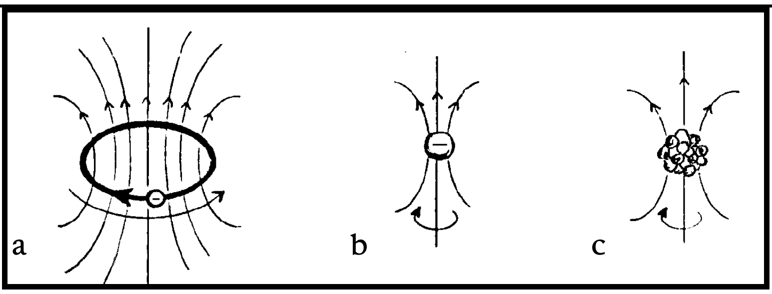


**Figure 6.13 Origin of atomic magnetic fields**

Fields due to (a) the orbital motion, (b)the spin of an atomic electron, (c) the spin of the nucleus.

### Ferromagnetism

In certain materials - in particular iron, cobalt, nickel and their alloys - the magnetic fields of adjacent atoms and nuclei interact so that in a small volume of the material (but still one that contains an enormous number of atoms) the magnetic fields from individual atoms line up. When an external magnetic field is applied these volumes or **domains** are aligned by the external field to produce a powerful magnetic effect. Such materials are said to be **ferromagnetic**. For some materials the domains remain aligned when the external field is removed, to form a **permanent magnet**.

### Diamagnetism and paramagnetism

For most materials the magnetic fields of adjacent atoms do not interact with each other, but only with an external magnetic field. For some materials the net magnetic field of each atom is zero. For each electron orbiting the atom in one sense there is one orbiting the atom in the other sense, so that the electron currents cancel. Similarly the spinning electrons have equal, but oppositely directed, magnetic fields. Such materials are said to be **diamagnetic** and are repelled from regions of strong magnetic field. Examples of diamagnetic materials are water and liquid nitrogen.

In the TV program the repulsion of water in an enclosed tube from a magnetron magnet is demonstrated. As the water is repelled the bubble moves towards the magnet, giving the impression that it is attracted to the magnet.

Some atoms do have a net magnetic field due to electronic and nuclear motion. However, because of thermal motion only a small fraction of these atoms can be aligned by an external magnetic field. Such materials are only weakly magnetic. They are said to **paramagnetic**, and are attracted towards a region of strong magnetic field. An example of a paramagnetic material is liquid oxygen.

## POST-LECTURE

### 6-9 WHY DO IRON FILINGS LINE UP ALONG FIELD LINES?

The way iron filings line up in a magnetic field can be explained superficially by assuming that the filings are magnetised by the magnetic field of the body under investigation, and the repulsive forces between like poles cause the filings to rotate so that they point along the field lines.

This is better understood if we imagine that the magnetism of the filings is due to circulating atomic currents. Imagine one such current loop. In a uniform field, the interaction with the field can be described as a pair of forces with the same magnitude but opposite directions and different lines of action (figure 6.14). This pair of forces produces a torque that rotates the current loop so that its final equilibrium position is normal to the field. As a result each magnetised filing points in the direction of the field.

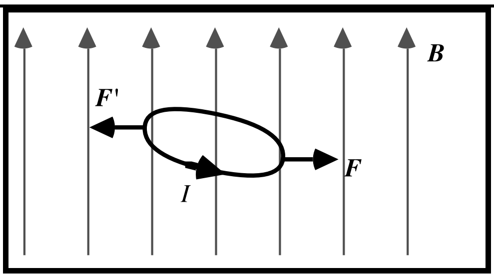


**Figure 6.14 Torque on a current loop in a uniform field**

If the field is non-uniform the two forces will not be exactly opposite (figure 6.15), so there will be a net total force which (in the absence of other significant forces) will produce an acceleration. Thus the filings will migrate along the field lines toward the region of stronger magnetic field. Note that there is still a torque which will twist towards the field direction.

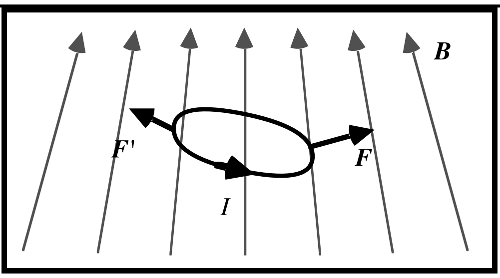


**Figure 6.15 Forces on a current loop in a non-uniform field**

## 6-10 Questions

**Q6.1** Sketch the magnetic field lines in the vicinity of two identical cylindrical coils, located with their axes coincident, and separated by twice the diameter of the coil, for the case where the same current flows around each coil in the *same sense*.

**Q6.2** Sketch the magnetic field lines in the vicinity of two identical cylindrical coils, located with their axes coincident, and separated by twice the diameter of the coil, for the case where the same current flows around each coil in the *opposite* sense.

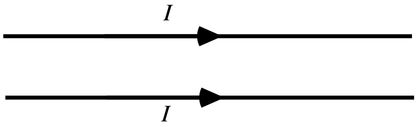
**Q6.3** Can an electron pass through a region of magnetic field without being deflected? Explain.

**Q6.4** The magnetic field in the radial gap of the permanent magnet of a loudspeaker is 0.050 T. The coil of this loudspeaker consists of 200 turns of wire wound with a mean diameter of 40 mm. Calculate the axial force acting on this coil when a current of 0.30 A flows through it.

**Q6.5** Calculate the magnetic field strength 10 m from a power cable carrying a current of 10 kA.

**Q6.6** Two cables each carrying a current of 10 kA, but in opposite directions, are separated by 100 mm. Calculate the (approximate) strength and direction of the magnetic field 10 m from the midpoint of this pair of cables, on a line joining the cables.

**Q6.7**



Each of two long parallel conductors carries a current $I$.

a) Describe the direction of the magnetic force on each wire.

b) If $I$ is doubled, by what factor does the force on each wire change?

**Q6.8** Look at figure 6.9 and figure 1.2. What is similar? What is different?

# E7 ELECTROMAGNETIC INDUCTION

## OBJECTIVES

### Aims

By studying this chapter you should get to understand the nature of the two kinds of electromagnetic induction, the differences between them and their common features. To demonstrate this understanding you should be able to explain the principles of electromagnetic induction and discuss examples of each of the two types.

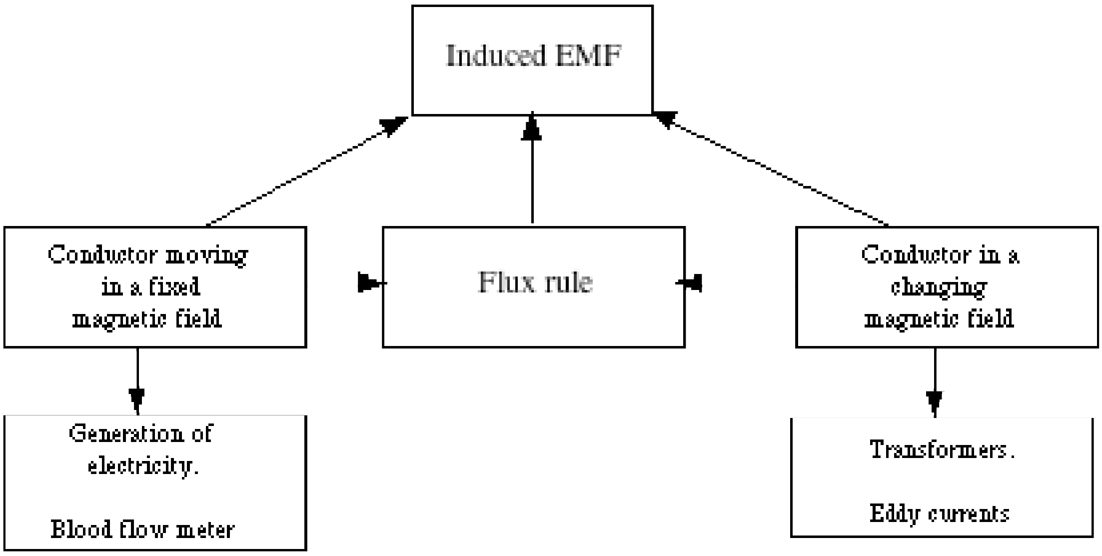
### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms

   *electromagnetic induction, EMF, induced EMF, motional EMF, magnetic flux, transformer, primary, secondary, eddy current.*
2. Describe and explain the two processes of electromagnetic induction
3. State and apply the result $\mathcal{E} = vBl$ for a moving conductor and specify the sense of the EMF.
4. Recall and use the formula for the EMF in a fixed plane conductor induced by a changing magnetic field perpendicular to the plane of the conductor .
5. State, explain and apply the flux rule (Faraday's law) relating induced EMFs of both kinds to rate of change of flux.
6. Describe and explain the origin of eddy currents.
7. Sketch, and explain the principles of operation of, a simple electric generator, an electromagnetic blood flow meter and a transformer.

### CONCEPT DIAGRAM



# PRE-LECTURE

## 7-1 INTRODUCTION - INDUCED EMF

Electromagnetic induction is a term used to describe the production of EMFs by two apparently quite different mechanisms: (1) the movement of a conductor through a region of space where there is a magnetic field and (2) the existence of a changing magnetic field in some region of space. In the first case, charged particles within a moving conductor experience magnetic forces which produce a charge separation which in turn creates a potential difference. In the second mechanism, a changing magnetic field creates an electric field - even in empty space. This induced electric field is not an electrostatic field because its field lines don't start and end on charges - but if some conducting matter is brought into the space the induced electric field can produce a charge separation and a measurable potential difference.

You should recall what is meant by EMF - see for example §5-2 in chapter E5. A source of EMF is something that is capable of giving energy to a system of charged particles by pulling the positive and negative particles apart. The value of the EMF is defined to be the energy supplied per charge. When the separated charges are in equilibrium and there is no current in the system, the potential difference produce by the separated charges is equal to the EMF.

It turns out that both of the effects called electromagnetic induction can be described by the same mathematical law, which we call Faraday's law after the person who probably did most to elucidate electromagnetic induction experimentally.

# LECTURE

## 7-2 VOLTAGE INDUCED IN A MOVING CONDUCTOR

When a length of wire is moved or deformed in a magnetic field, a voltage appears between the ends of the wire. The existence of the voltage depends essentially on the movement of the wire: when the wire is not moving there is no voltage. However the factors which determine the magnitude and sense of the voltage are not obvious. They are explored by means of a number of experiments in the video lecture.

### Demonstrations

A brass rod is driven at a steady speed between the poles of a strong magnet. The rod, the magnetic field and the direction of motion are all perpendicular to each other. When the rod is moved through the magnetic field there is an **induced voltage**; when the motion stops the voltage disappears.

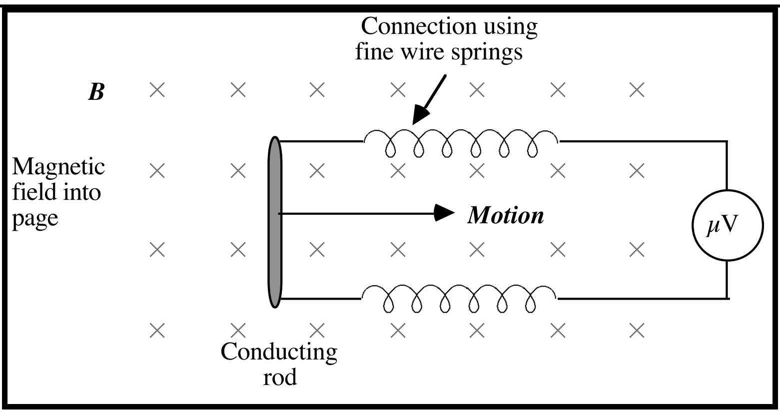


**Figure 7.1 Demonstration of motional EMF**

When the velocity is reversed the same voltage, in the opposite sense, appears. When the conductor is then replaced with one of half the length, and it is driven through the magnetic field at the same velocity as before, the voltage registered is half the previous value. Careful measurements confirm that for the same speed and magnetic field the induced voltage is proportional to the length of the conductor.

For the next experiment, the conducting rod is attached to a pendulum, and the induced voltage is displayed on a chart recorder. As the rod moves backwards and forwards through the magnetic field, the voltage indicated by the chart recorder has its greatest magnitude when the speed is greatest and is zero at the extremes of the swing when the speed is zero. As the pendulum loses energy, its speed at the middle of the swing decreases and the amplitude of the induced voltage falls correspondingly.

The experiment is repeated with a shorter pendulum using the same initial displacement. The speed and corresponding induced voltage are greater. Careful measurements establish that the magnitude of the induced voltage is proportional to the speed of the conductor, and the sign of the voltage reverses when the conductor moves in the opposite direction.

The same pendulum apparatus is used to show that the induced voltage depends on the magnetic field strength. To do this, the pole pieces of the magnet are removed, increasing the gap and reducing the magnetic field strength to about half its previous value. When the pendulum is released with the same amplitude of swing as before, the chart recorder shows that the amplitude of the induced voltage is reduced. Other quantitative experiments show that, for a given speed, the induced voltage is proportional to the magnitude of the magnetic field.

### The voltage induced in any moving conductor

Combining the results of all these and other experiments yields the result that the voltage $V$ induced in a conductor of length $l$ moving with speed $v$ perpendicular to the length of the conductor and perpendicular to the magnetic field is

$$V = vBl$$

where $B$ is the strength of the magnetic field. If the velocity makes an angle $\theta$ with the field, the induced voltage is reduced by a factor of $\sin\theta$, so

$$V = vBl\sin\theta$$

The potential differences appearing across the conductors in these experiments are a consequence of the **induced EMF** within the conductor.

### Theory of motional EMF

Consider a metal rod moving through a magnetic field, like the one used in the demonstrations.

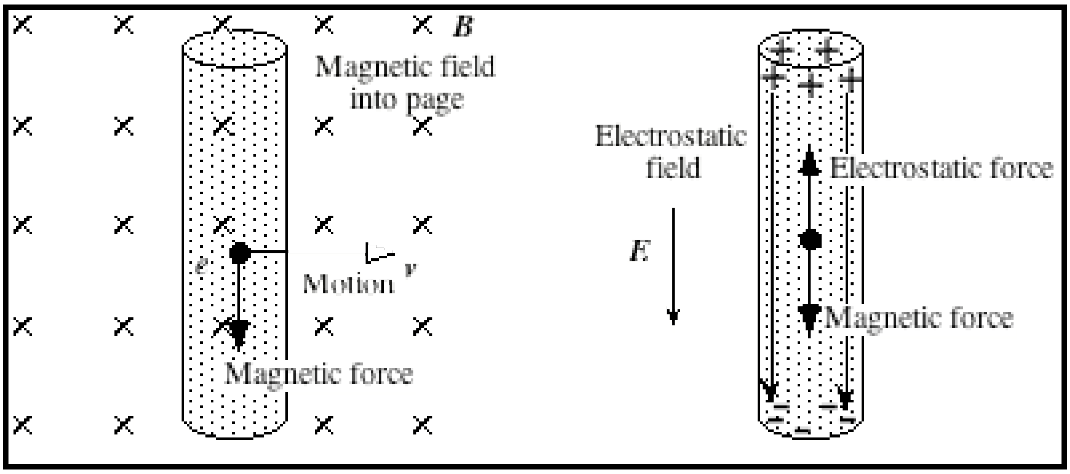


**Figure 7.2 Origin of motional EMF**

The conduction electrons in the rod, which are moving through the magnetic field, experience a magnetic force which is perpendicular to both their motion and the field. This force pulls electrons towards one end of the rod (left side of figure 7.2). Electrons will start to move, under the influence of the magnetic force and other interactions with particles in the rod. The effect of that movement is to produce a charge separation which immediately creates an electrostatic field that tries to pull the electrons back (right hand diagram in figure 7.2). An equilibrium is soon established in which there is a balance between the electrostatic and magnetic forces. Consider an electron somewhere in the rod (not at the ends). The electrostatic and magnetic forces on it must be balanced, so $eE = evE$. Cancelling the charge $e$ you can see that the magnitude of the electrostatic field is

$$E = vB.$$

There is a potential difference $V$ associated with the electrostatic field. Since the field is equal to the (negative) potential gradient we can say that the potential difference ($V$) between two points separated by a distance $l$ and the magnitude of the average electrostatic field are related thus:

$$E = \frac{V}{l} \ .$$

By comparing the two expressions for the electric field we get a relation for the voltage, $V = vBl$. Since we are considering a case where there is no current, the potential difference must be equal to the EMF which caused it so we can say that

$$\mathcal{E} = vBl \ . \qquad \text{... (7.1)}$$

Remember that this relation applies only to long thin objects with their long axis, their velocity and the field all mutually perpendicular.

## 7-3 APPLICATIONS OF MOTIONAL EMFS

### Generation of electrical power

The design of electrical generators, which depend on EMFs induced in conductors moving through regions of magnetic field, is a highly specialized branch of engineering. Demonstrations in the lecture illustrate how the simple idea of moving a wire near a magnet has been exploited and refined to produce efficient generators of hundreds of megawatts of electrical power.

### An electromagnetic blood flow meter

In studies of cardiovascular physiology, one of the most fundamental quantities is the flow rate of arterial blood. One method for measuring blood flow rate in experimental animals was developed in Australia, and is now used by physiologists world-wide. It uses the EMF induced in the flowing blood.

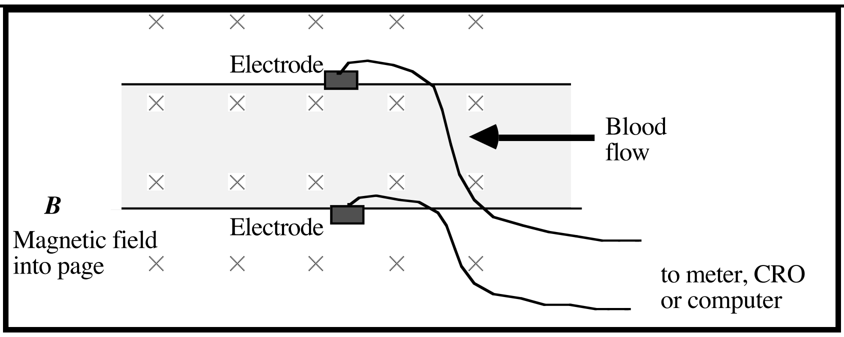


**Figure 7.3 Principle of the electromagnetic flow meter**

A magnetic field (of about $10^{-3}$ T) is set up across the artery, and connection is made to two small electrodes in contact with the artery (the line joining the electrodes is perpendicular to the magnetic field). From equation 7.1 the average speed of the blood is thus

$$v_{\text{blood}} \approx \frac{V}{Bd}$$

there being some uncertainty as the speed varies radially in the artery.

The probe of such an instrument consists of a plastic moulding which contains coils to generate the magnetic field and electrodes to sense the EMF. See figure 7.4.

The probes are made in various sizes, as the artery has to fit properly to make good contact with the electrodes. The probes can be calibrated quite accurately: the observed EMF is readily converted to blood flow rate in the practical unit of mL.min$^{-1}$. The instrument can either show the mean flow rate on a meter, or display the variation during the pulse on an oscilloscope or chart recorder.

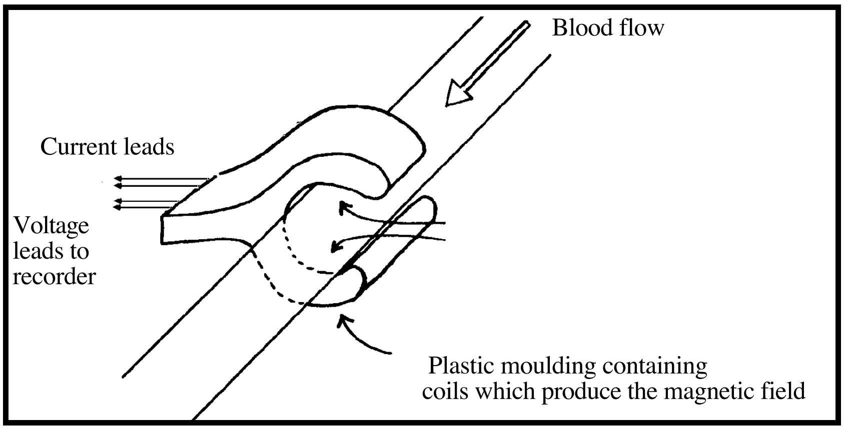


**Figure 7.4 Construction of the blood flow meter**

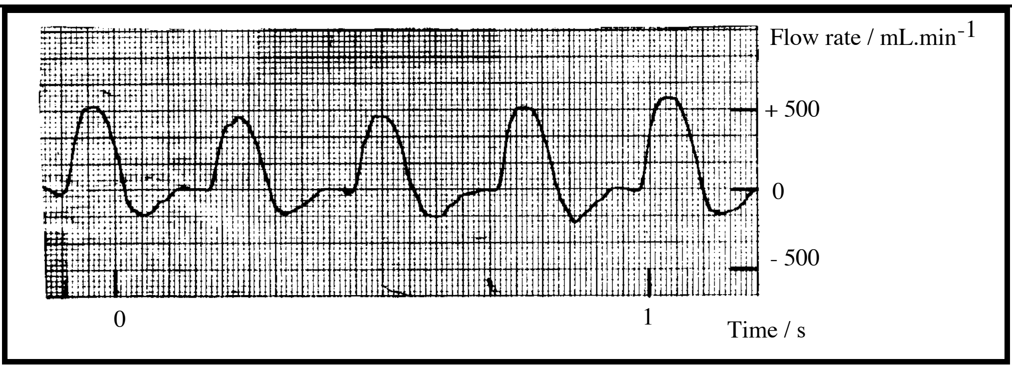


**Figure 7.5 Recording of blood flow in the aorta of a small mammal**

## 7-4 VOLTAGES INDUCED BY CHANGING MAGNETIC FIELDS

### Moving coils and moving magnets

When a coil of wire is moved near a magnet, a motional EMF is induced in the coil. This has already been explained in terms forces on moving charges. However if the coil is held stationary, and the magnet is moved (in the opposite direction with the same speed) exactly the same voltage is induced, even though the conductor is at rest. That is a bit of a puzzle because the explanation of the induced EMF given in §7-2 depends explicitly on having a moving conductor.

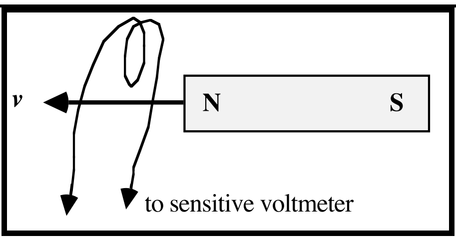


**Figure 7.6. EMF induced by a changing magnetic field**

Nevertheless the effect is real. It turns out that it is the *relative* velocity of the coil and magnet that matters. However, think of this second experiment from the point of view of an observer at the coil who cannot see the magnet. That observer would relate the induced EMF to an increasing magnetic field at the coil. This argument suggests (but does not prove) that EMFs can be produced by a changing magnetic field.

### Demonstrations: experiments with coils and changing currents

A digitally controlled function generator is used to produce a current which varies with time in a controllable way. The current is passed through a coil of about 100 turns, and is displayed on the upper pen of a chart recorder. A second, smaller diameter, coil with a similar number of turns is placed coaxially inside the larger coil. This inner coil is connected to the lower pen of the chart recorder.

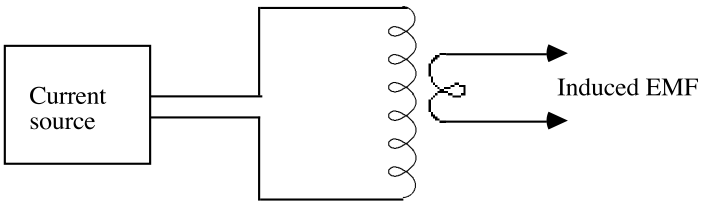


**Figure 7.7 Apparatus for studying EMFs from changing fields**

By adjusting the current *I* in an irregular manner by hand, it is shown that an EMF is induced in the inner coil when, and only when, the current on the outer coil is changing. **The changing magnetic field of the outer coil induces an EMF in the inner coil.**

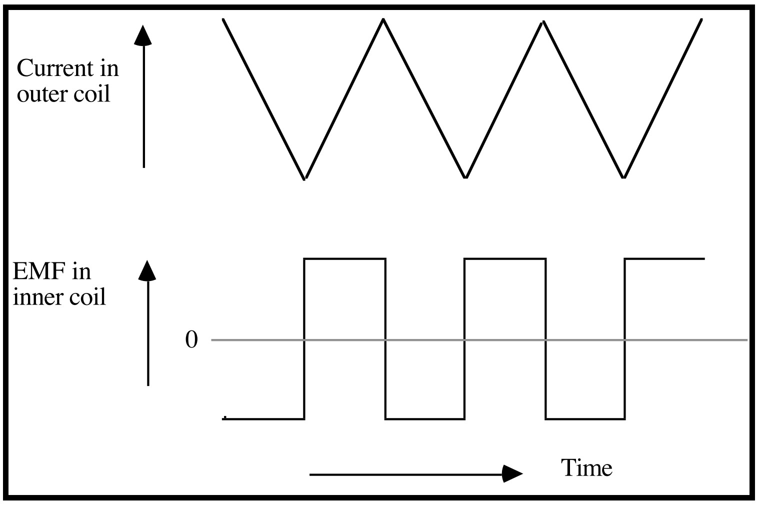


**Figure 7.8 Changing current and the induced EMF**

The function generator is next set to increase steadily, and then decrease the current, between fixed limits. The experiment shows that while the current in the outer coil is rising linearly, the induced EMF remains constant. When the current reverses, the induced EMF reverses.

Runs with the current changing twice and four times as rapidly show corresponding increases in the induced EMF. A further run in which the current is held briefly constant at its extreme values confirms that it is the rate of change of current, rather than the current itself, which determines the induced EMF.

These demonstrations show that the induced EMF in a coil is proportional to the rate of change of the magnetic field:

$$\mathcal{E} \propto \frac{\mathrm{d}B}{\mathrm{d}t} .$$

To produce an easily measured induced EMF in a single-turn coil, a more rapidly changing current is passed through the large coil. A sinusoidally varying current (at about 400 Hz) is used, and the output waveform is displayed on a CRO.

With this arrangement of the apparatus, three points are demonstrated.

- The voltage induced in a coil depends on the coil area
- The sense of the measured voltage changes when the coil is turned over.
- The induced voltage is proportional to the number of turns in the coil.

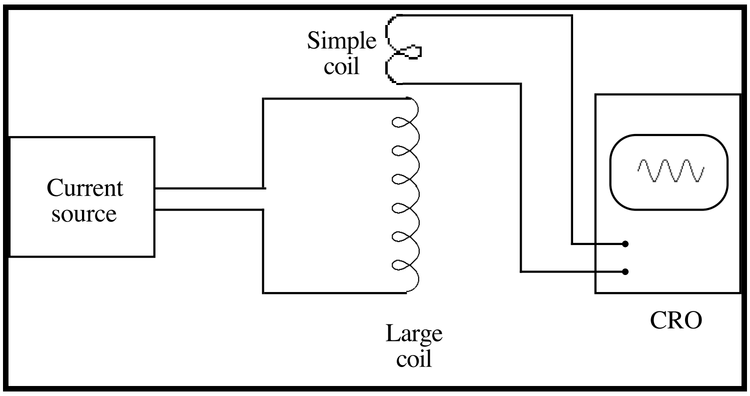


**Figure 7.9 Apparatus to display EMFs**

All these experiments are summarised by saying that if there is a uniform changing magnetic field perpendicular to a plane coil, the EMF induced in the coil is proportional to the product of the number of turns in the coil, its area and the rate of change of magnetic field inside the coil.

$$\mathcal{E} = \text{const} \times NA\,\frac{\mathrm{d}B}{\mathrm{d}t}\ .$$

In fact, as is discussed in the post-lecture, the constant in the above expression is 1 so

$$\mathcal{E} = NA\,\frac{\mathrm{d}B}{\mathrm{d}t}\ . \qquad \text{... (7.2a)}$$

## 7-5 THE FLUX RULE

For a single turn flat coil perpendicular to a uniform but changing field the induced EMF can be written

$$\mathcal{E} = \frac{\mathrm{d}(AB)}{\mathrm{d}t} \qquad \text{... (7.2b)}$$

where $AB$, the product of the coil area and the component of the magnetic field perpendicular to the area, is called the **magnetic flux** ($\Phi$) through the coil.

To define magnetic flux in general, first imagine a surface, small enough so that you can say that the magnetic field doesn't vary over the surface. Find the component of the magnetic field perpendicular to the surface; that gives $B\cos\theta$, where $\theta$ is the angle between the field and a direction perpendicualr to the surface. The flux through the surface is the product of that component and the surface area, $\Delta A$:

$$\Delta\Phi = B\,\Delta A\cos\theta \qquad \text{... (7.3)}$$

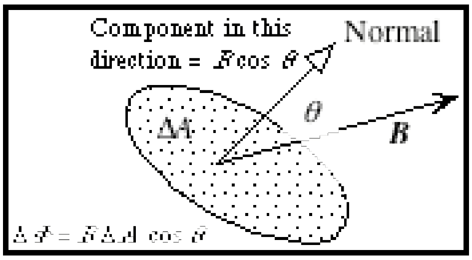


**Figure 7.10 Magnetic flux**

To find the flux through a larger surface, you just repeat the process and add (or integrate) the flux for each piece. The SI unit of magnetic flux is the called the weber (Wb).

To visualise the meaning of magnetic flux, think of it as the number of magnetic field lines passing through the area.

The equations (7.2a,b) for the EMF induced in a flat coil represent a special case of the general law, known as the flux rule or Faraday's law:

$$\mathcal{E} = N\,\frac{\mathrm{d}\Phi}{\mathrm{d}t}\ ; \qquad \text{... (7.4)}$$

the EMF induced in a coil is equal to the rate of charge of magnetic flux through one turn, multiplied by the number of turns in the coil.

This way of thinking is very powerful because it provides the link between EMFs induced in moving conductors and EMFs induced by changing magnetic fields.

## 7-6 APPLICATIONS OF EMFS FROM CHANGING MAGNETIC FIELDS

Devices, instruments and machines which depend on induction of voltages by changing magnetic fields are commonplace. Some examples are: metal detectors, traffic control sensors, hi-fi cartridges, video and audio tape recorders, cash dispensing machines, electric guitars and microwave ovens. Two examples, transformers and eddy currents, are considered in a little more detail.

### Transformers

The simplest **transformer** consists of two separate coils of wire, insulated from each other, both wound on the same iron core. In circuit diagrams a transformer is represented with the coils and core separated for clarity as in figure 7.11.

An alternating voltage applied to one winding, the **primary**, causes an alternating current, thereby producing an alternating magnetic flux in the core. This alternating flux in the core induces a voltage in the **secondary** winding. The secondary voltage depends on the ratio of the number of turns in the primary and secondary coils. For an ideal transformer,

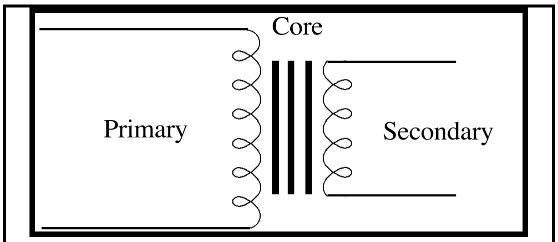


**Figure 7.11 A transformer**

$$\text{secondary voltage} = \text{primary voltage} \times \frac{\text{no. of secondary turns}}{\text{no. of primary turns}} \cdot$$

Physically, transformers range from the huge oil-cooled ones used to step up the voltage of electrical power for economical transmission to tiny air-cored ones used in the tuned circuits of radio receivers. Apart from their function in changing AC voltages and in matching power sources to their loads, transformers play an important role in supplying electrical energy to devices without any direct conducting connection. Such isolation may literally be of vital importance when electrical equipment is used in treating and monitoring patients in hospital.

### Eddy currents

In a changing magnetic field, voltages will be induced not only in coils, but in any conductor in the field. As a result loops of current called **eddy currents** will be set up in the conductor. Often they are a nuisance; for example the cores of transformers and motors have to be made of insulated layers (laminations) of iron to prevent eddy currents in the core, and consequent waste of energy.

Once an eddy current exists, it experiences a force in the magnetic field, producing striking and useful effects. In the video lecture, a swinging sheet of non-magnetic metal (aluminium) is rapidly brought to rest by a magnet. This effect, called **eddy current damping**, is produced by the magnetic force on the eddy currents.

The same mechanism explains the operation of one common type of speedometer. A rotating magnet induces eddy currents in an aluminium disk. The magnetic force on these currents produces a torque on the disk, and turns it. A spring exerts a restoring torque on the disk so that the angular deflection is proportional to the angular velocity of the magnet.

## POST-LECTURE

### 7-7 THE FLUX RULE

In this chapter we have considered separately two distinct types of electromagnetic induction. While these two phenomena appear to be physically different they are closely related. Indeed one law, known as the **flux rule** can be used to correctly calculate the EMF, whether it is induced by a changing flux linkage or a conductor moving in a magnetic field. The basis for the rule is worth studying because it gives a deeper understanding of electromagnetic induction than is possible in the video lecture.

Consider a pair of conducting rails connected at one end to form a U-shape. The plane of this conductor is perpendicular to a uniform magnetic field (figure 7.12).

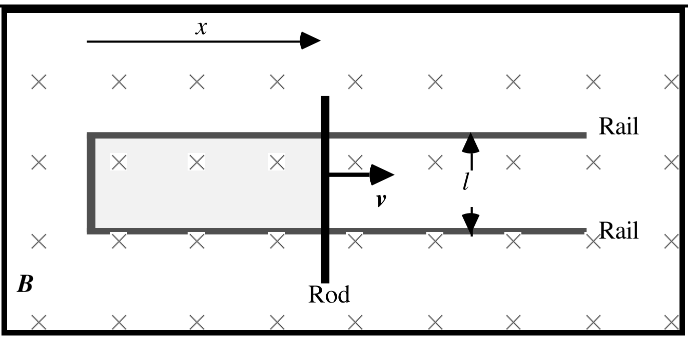


**Figure 7.12 Motional EMF and the flux rule**

The crosses indicate that the magnetic field, magnitude $B$, is directed down into the page. A conducting rod (solid line) rests on the parallel conductors, making contact with them.

Now think what happens when the conductor is moved at velocity $\boldsymbol{v}$ to the right. We calculate the EMF induced in the shaded conducting loop by two methods.

1) The rod is moving so the EMF induced in it by its motion across the magnetic field is

$$\mathcal{E} = Blv$$

and this is thus the EMF induced in the loop.

2) The conducting loop around the shaded area is a single turn coil in the magnetic field $B$. The magnetic flux linking this coil is

$$\phi = Bxl$$

where $xl$ is the area enclosed by the conducting loop.

The EMF can be calculated from the rate of change of this linking flux:

$$\mathcal{E} = \frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}(Bxl)}{\mathrm{d}t} .$$

Since $B$ and $l$ are constant, this yields

$$\mathcal{E} = Bl\frac{\mathrm{d}x}{\mathrm{d}t} = Blv .$$

In this example, the induced EMF can be worked out either from the EMF in a moving conductor (equation 7.1) or from the changing flux linkage (equation 7.4). This leads to a statement of the flux rule for calculating induced EMFs: **the induced EMF is equal to the rate of change of magnetic flux linking the circuit.**

Remember that when we are dealing with coils with many turns ($N$ say) the flux links each turn, so the EMF is $N$ times that for a single turn.

## 7-8 QUESTIONS

**Q7.1** In the video lecture, one demonstration shows that a voltage of 50 $\mu$V is induced between the ends of a rod of length 40 mm when it moves at 10 mm.s$^{-1}$ perpendicular to its own length between the pole pieces of a powerful magnet. Calculate the strength of the magnetic field.

**Q7.2** Sketch the essential parts of an electromagnetic blood flow meter. One such instrument generates a transverse magnetic field of $1.0 \times 10^{-3}$ T. When it is used on an artery of internal diameter 5.0 mm, the induced EMF is 2.5 $\mu$V. Calculate the blood flow rate assuming a plane velocity profile.

**Q7.3** A coil is moved at a constant velocity towards one pole of a bar magnet. When the coil is 0.10 m from the magnet, the EMF induced in the coil is 40 mV. The coil has 20 turns and area $1.0 \times 10^{-5}$ m$^2$. What EMF would be induced in the following experiments?

- The coil is held stationary, and the magnet is moved with the same velocity away from the coil at the moment when separation is again 0.10 m.
- Both coil and magnet are moved in the same direction, at constant separation of 0.10 m.

When the EMF of 40 mV is induced in the stationary coil, what is the average rate of change of magnetic field at the coil?

**Q7.4** Explain how the strength of an alternating magnetic field can be measured using a small coil having a large number of turns (a search coil).

Such a coil has 1000 turns and a diameter of 20 mm. In a 50 Hz magnetic field an AC voltage of 1.0 V (peak-to-peak) is observed. What is the amplitude of the 50 Hz magnetic field?

**Q7.5** Briefly explain two uses for transformers. If the magnetic flux in a particular transformer is changing at the rate of 1.0 T.m$^2$.s$^{-1}$, what EMF would be induced on a winding of 500 turns?

**Q7.6** One type of metal detector works by inducing eddy currents in the treasure and then detecting the magnetic field of these currents. Which of the following do you think it would discover most readily, given that gold has a higher resistivity than silver:

1) 1 kg of gold Spanish doubloons in an old sack
2) a 1 kg lump of gold,
3) a 1 kg lump of silver?

Give your reasons.

**Q7.7** A flat sheet of copper is placed between the poles of a strong magnet. When somebody tries to pull it out, there is a strong force resisting the motion. Explain what is happening. How can the force be reduced?

# E8

# APPLICATIONS

## OBJECTIVES

### Aims

From this chapter you should gain a basic understanding of two important applications of electrical principles to the human body - the study of the function of the heart and electric shock. You should also develop an awareness of the principles of electrical safety and electrical wiring.

### Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

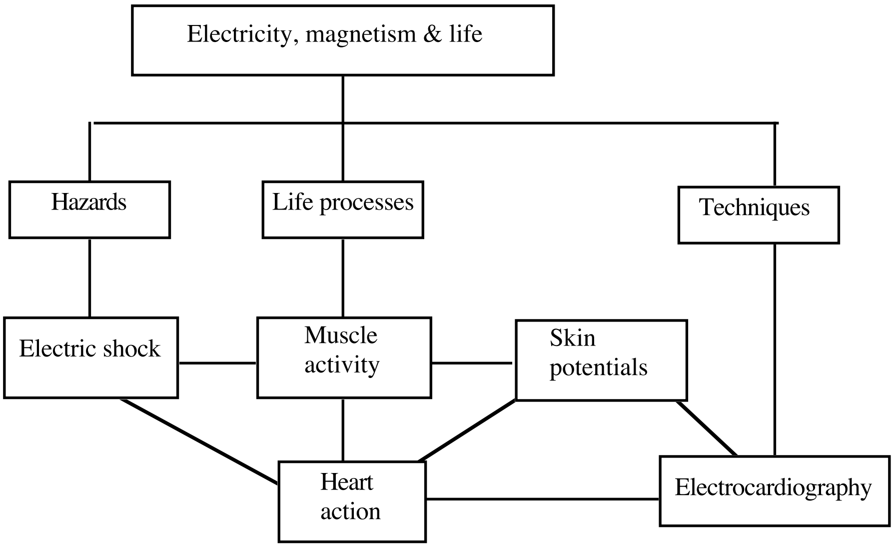
1. Explain, interpret and use the terms:

   *polarisation, depolarisation, electric shock, shocking current, active lead, neutral lead, earth, earthing, double insulation, microshock.*

2. Describe the electrical behaviour of the heart and explain the general principles of electrocardiography.
3. Describe the physiological effects of alternating currents passing from limb to limb in the human body.
4. Describe emergency and first-aid steps for victims of electric shock.
5. Describe the multiply-earthed neutral (MEN) system of electric power distribution.
6. Describe examples of possible active-to-earth electric shock accidents, discuss the conditions under which such accidents might be fatal, and describe precautions for the prevention of such accidents.
7. Describe the Australian colour code and wiring conventions for three-pin power plugs and sockets.

## PRE-LECTURE

### CONCEPT DIAGRAM



## LECTURE

### 8-1 ELECTROCARDIOLOGY

Electrocardiology is a technique widely used in medicine and veterinary science for diagnosing the condition of the heart from its electrical activity. Since the body is a reasonably good conductor of electricity, currents generated by the heart produce potential differences on the skin. By attaching electrodes at a number of standardised points, these potentials, and hence the activity of the heart can be monitored. The purpose of this section of the lecture is to examine the physical principles underlying this vital clinical technique.

#### Individual heart-muscle cells

The heart is a special kind of muscle which, unlike the smooth and striated muscle found elsewhere in the body, can continuously contract and relax; this is the heartbeat. In practice, the heart's muscle cells are elongated. As we are concerned only with their electrical behaviour it is simpler, and adequate, to represent them as spherical. In the relaxed (i.e. non-contracted) state, a heart muscle cell is bounded by an electric double layer at the cell wall (figure 8.1). The cell is said to be **polarised**.

Electrodiffusive processes maintain a potential of -90 mV on the cell interior, relative to the surroundings. There is an electrostatic field in the cell membrane, between the layers of positive and negative charge. However (in accordance with Gauss's law) there is no electric field produced outside the cell. Consequently, between beats, when the whole heart is relaxed, it produces no external electric field. You may recall from chapter E4 that the membrane's sodium pump maintains a concentration of sodium ions outside the cell that is much greater than that inside.

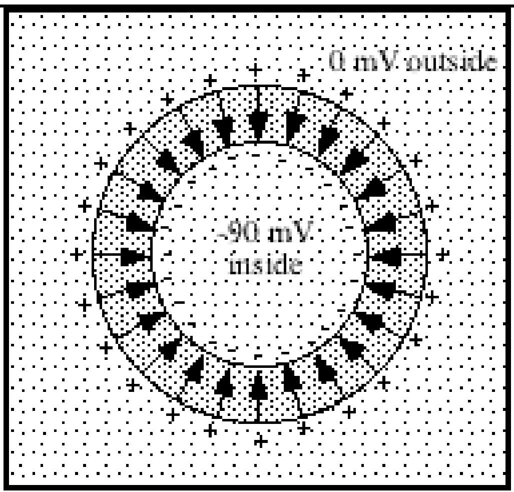


**Figure 8.1 Idealised view of a heart-muscle cell**

During the heartbeat, the cells are progressively depolarised (i.e. the charge layers disappear) in a coordinated way. The permeability of the cell membrane to sodium ions suddenly but briefly increases. Sodium ions rush into the cell, driven by both the concentration gradient and the electric field within the membrane. The inside of the cell becomes positive and the outside negative. There is movement of ions both inside and outside the cell as the charges are redistributed.

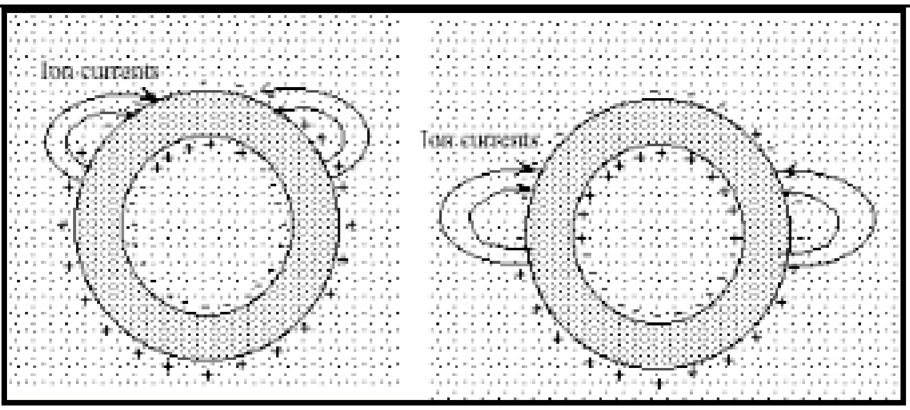


**Figure 8.2 A depolarising cell**

Associated with these ion currents there is an electric field and associated potential differences at places outside the cell, where previously there was no field. Figure 8.2 represents a cell which is partly depolarised, together with the transient ion currents. The combined effect of the simultaneous depolarisation of many cells produces the external electrical activity of the heart. Once a cell is fully depolarised, there is again no external electric field, and therefore once again the cell makes no contribution to the external electrical activity of the heart. Following the wave of depolarisation the sodium permeability returns to normal and the sodium-potassium pump restores each cell to its normal resting state.

Neighbouring cells in the heart wall depolarise in a coordinated manner, so that their external electric fields add up to produce a large scale electric field pattern, which changes as the depolarisation progresses.

## The heartbeat

In their relaxed state, between beats, all the cells of the heart are polarised, and there is no external electric field. Muscle contraction, and the associated wave of cell depolarisation, starts at the sino-atrial node (top right of the heart in figure 8.3).

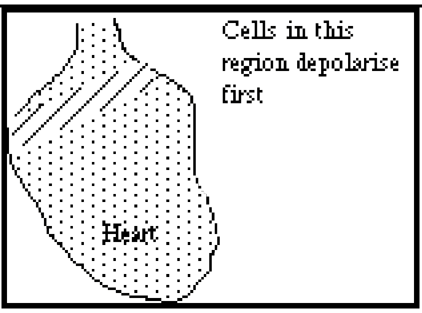


**Figure 8.3 A region of depolarisation**

The region of contraction and depolarisation initially sweeps through the atria (the pumping chambers at the top of the heart); the combined effect of all the partially depolarised cells in this region generates an external electric field.

The heart as a whole behaves like a dipole, negative above and positive below, with an upwardly directed electric field in the regions of the body around the heart (figure 8.4).

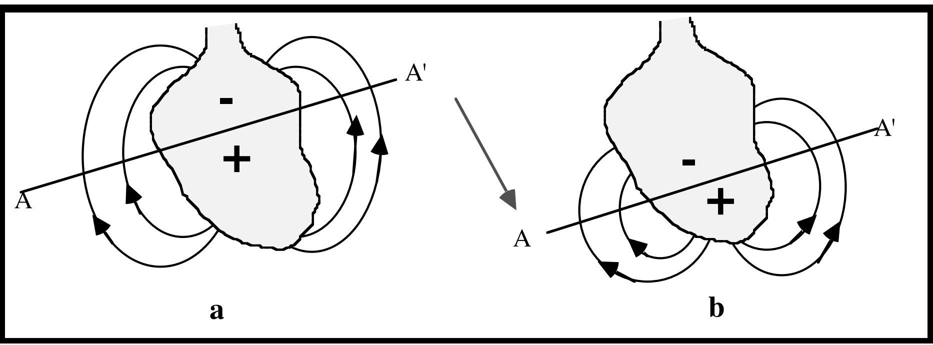


**Figure 8.4 Electric field generated by depolarisation**

Since the individual cells produce an external field *only* while they are in the process of depolarising, the instantaneous value of the external field is determined *only* by those cells in the depolarising region (the plane AA' in figure 8.4a). It is this feature which makes electrocardiograph (ECG) diagnosis so valuable.

The wave of depolarisation pauses briefly after passing through the atria, then continues to sweep downwards and to the left as the ventricles contract. At the stage shown in figure 8.4b, about 250 ms after the initiation of the beat, the main contraction of the ventricles is occurring. The electric field (and the ECG signal) reaches its greatest strength, and gives us information about the ventricular region of the heart. At other times during the beat, signals give information about other regions depending on the position of the plane of depolarising cells.

### The heart as a battery in the chest

During the heartbeat, the heart as a whole behaves like an electric dipole which varies markedly throughout the cycle, but which always has the same polarity (the upper region of the heart is always negative). In understanding its external electrical effects, the heart can be thought of as a variable battery, located inside the chest with its negative electrode pointing up and to the right of the person (figure 8.5).

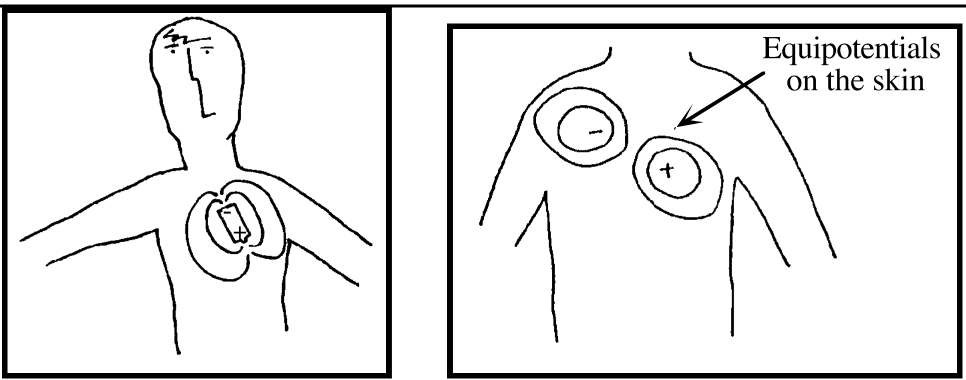


**Figure 8.5 The heart acting as a battery**

Currents generated by the heart (battery) exist in the resistive tissues of the body and give rise to potential differences over the surface of the skin. At the peak of the heart's electrical output, the potential distribution over the thorax looks like the right-hand part of figure 8.5. The lines are equipotentials drawn at intervals of about 0.25 mV. The potentials change during the pulse, as the strength and position of the heart's electric dipole change.

### The electrocardiograph

By attaching electrodes to the skin at standard positions (up to 12) and recording the potentials during the pulse, the main features of the surface potential distribution, and hence of the electrical activity of the heart, can be determined. Since, as we saw above, any given cell produces an external electric field only while it is in the process of depolarising, each instant on an ECG trace corresponds to a particular region of the heart. Figure 8.6 shows the principle features of an ECG trace (where the ECG potential is measured between electrodes attached to the upper right and lower left regions of the chest) and the region of the heart corresponding to each feature.

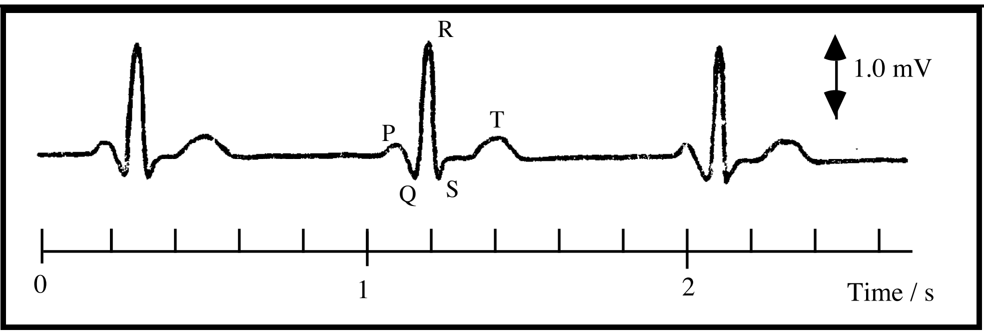


**Figure 8.6 A typical ECG trace**

Electrodes attached to top right and bottom left of a patient's chest. P: depolarisation of atria; QRS: depolarisation of ventricles; T: repolarisation of ventricles.

### More about cardiology

In the lecture we discuss the usual clinical electrocardiograph using external electrodes and a chart recorder. In intensive care, internal cardiac electrodes are often used to obtain more direct and detailed information. In research, but not generally clinically, the ECG recordings may be processed to give a three-dimensional display of the dipole moment of the heart (this is the so-called vector cardiograph). With minor variations of technique and interpretation, electrocardiography is also used in veterinary practice.

## 8-2 ELECTRIC SHOCK

Fatal electric shock occurs when a sufficiently large electric current flows through the body. The danger arises because a fraction of such a current flows through the heart and may disrupt the cardiac cycle. In the most common domestic and industrial electric shock accident, the contact with live conductors of the electricity mains causes current to flow through the body, typically from arm to arm or from arm to leg. In this section of the lecture we discuss the effects of such electric currents, how they can arise in accidental situations and the principles of electrical safety.

### The physiological effects of electric current

Suppose the body is accidentally connected to a circuit and a current (*I*) flows from hand to hand (similar results would ensue for the same current from hand to feet, head to bottom, etc.).

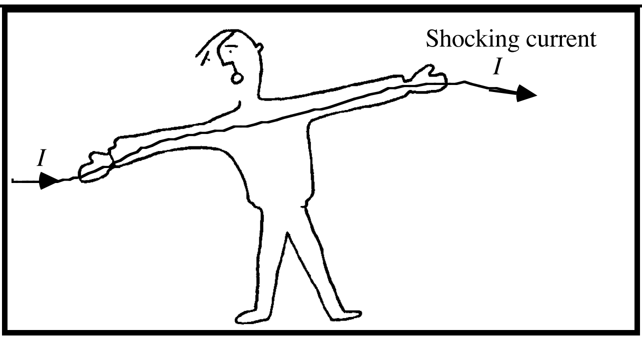


**Figure 8.7 Electric shock**

The effects of such a current are shown in table 8.1. The effects do not suddenly occur at definite values of current. Variation occurs with age, sex and health of the victim and with the path taken by the current in the body. There are also differences between the effects of alternating and direct currents.

| Shocking current | Effect |
|---|---|
| Less than 1 mA | No observable effect. |
| ~1 mA to ~10 mA | Tingling sensation. |
| ~10 mA | 'Let go' current.<br>Muscular paralysis. |
| ~10 mA to ~100 mA | Laboured breathing. |
| ~100 mA | Breathing stops.<br>Ventricular fibrillation. |
| ~1 A to ~10 A | Thermal damage to tissue. |

**Table 8.1 Somatic effects of electric current**

***Notes***

- For currents less than 100 mA effects are usually reversible when the current is interrupted.
- Typically a 50 A, 10 ms pulse is used to defibrillate the heart

If a current of less than about 1 mA is passed through the body, there is generally no sensation and apparently no significant physiological effect. As the current is increased to a few milliamperes, a tingling sensation is felt, becoming more and more uncomfortable as the current is increased.

At about 10 mA muscles contract and are held contracted; in particular the fingers clamp on to a conductor, and it is impossible to open the hand. The term *let go current* means the highest current at which one can still let go. It is important to note that a current just above the let go current value is usually harmless. When the current is broken, the victim is shaken but alive.

As the current rises above 10 mA, breathing becomes laboured and may cease. The victim may still recover spontaneously if the current is promptly stopped.

A current of about 100 mA sustained for a second or so is all that is needed to switch the heart over from its normal pumping rhythm to a useless random twitching action known as **ventricular fibrillation**. The heart action does not recover spontaneously when the current is broken.

Currents much greater than 100 mA can also cause internal and skin burns. Paradoxically, brief currents of > 1 A may be less dangerous than lower currents, as instead of putting the heart into ventricular fibrillation, they are sufficient to clamp the whole heart muscle simultaneously, and when the current is turned off, the normal heart beat may resume of its own accord. Indeed, currents of about 1 A are used clinically to defibrillate the heart.

### First aid for electric shock victims

Anyone working with electrical equipment should be familiar with the technique of cardio-pulmonary resuscitation (CPR). Courses available to the public are run from time to time by hospitals and first aid organisations. Briefly, if someone is struck by electric shock:

1) interrupt the current;
2) if breathing has stopped, apply mouth-to-mouth breathing;
3) if there is no pulse, apply full CPR;
4) get an ambulance quickly.

It is vital to realise that if the heart has gone into ventricular fibrillation it will not restart spontaneously. The victim will not survive unless CPR is started promptly and continued until expert medical treatment, including defibrillation, is applied.

### Live or active mains leads

In this and the following paragraphs we consider how the body can sustain a dangerous shock by accidentally becoming part of a mains power circuit. To do this, it is first necessary to understand the principles of mains power wiring. A domestic or industrial (single phase) supply can be represented in simplified form as in figure 8.8.

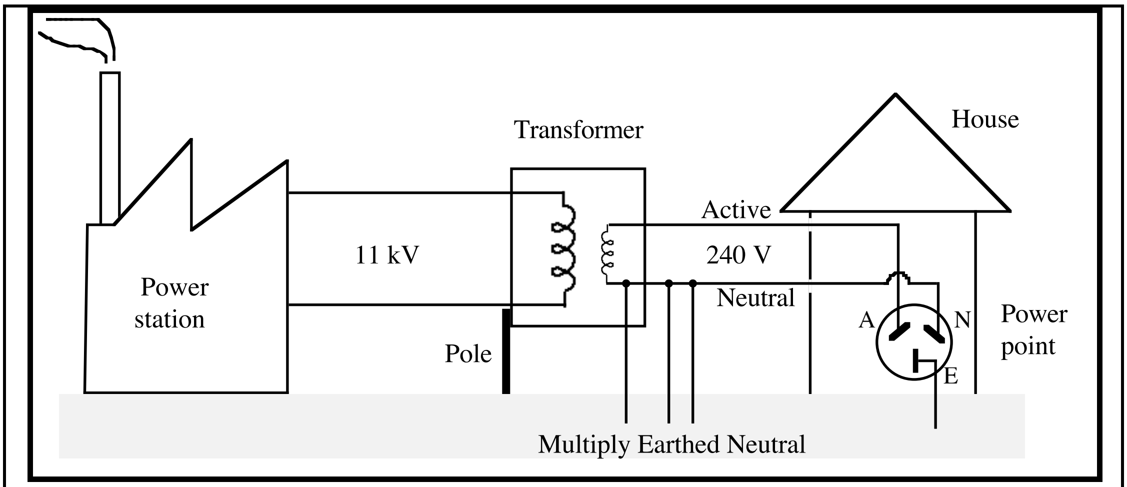


**Figure 8.8 Electrical power network**

Power from the generator is transmitted at high voltage (typically 11 kV AC) to various substations and local transformers where it is stepped down to 240 V for general use. You can see the local transformers on poles in the street. At the transformer, one lead, known as the **neutral** (N) is connected to earth via a thick cable with one end buried in the ground. The other wire from the transformer, the **active** lead (A) is thus at an average potential of 240 V relative to the ground, i.e. to our surroundings. Active and neutral conductors are connected, albeit via a complex distribution network, to the A and N points of the user's outlet sockets. The user's **earth** (E) point is connected as directly as possible to the ground close to the house. Since the neutral wires in the power lines carry large currents and since they have finite resistance (ideally the resistance would be zero) they cannot be at exactly the same potential at all places along the wire. So, as shown in figure 8.8, the neutral is usually connected to ground at several other points between the transformer and the houses (this is the so-called MEN or multiply-earthed neutral wiring system) in order to keep the neutral lead at a potential near ground (zero). This ensures that only the active lead is at a high potential which is less dangerous than having two high voltage wires. Note, however, that the neutral lead cannot be exactly at earth potential at all places.

### Active-to-ground shocks

In normal use, current goes to the electrical equipment (e.g. toaster) via the active wire and back to the transformer via the neutral. The current reverses every 10 ms (for 50 Hz AC supply). However, because the neutral is earthed, the current can return via the earth rather than the neutral. For example, an unwary breakfaster, touching the live end of the toaster element with a knife can get a shock.

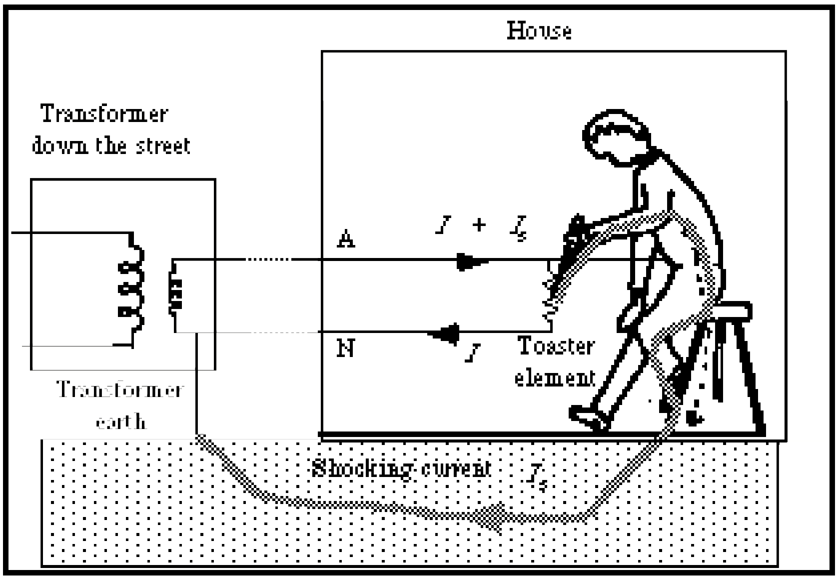


**Figure 8.9 Electric shock from a toaster**

In addition to the normal current, $I$, through the toaster there is an additional shocking current $I_S$ through the active lead, through the knife, hand, arm, body, chair, floor and back to the neutral via the earth (see diagram above). How dangerous such a shock is depends on the current, which in turn depends on the resistance in the path of the shocking current, i.e. the resistance provided by the person, floor, chair, etc.

### Calculation of the shocking current

In the example above, the average potential between active and earth is 240 V. The resistance in the path of the current through the breakfaster is made up as follows:

$$R \text{ (total)} = R \text{ (mains supply)} + R \text{ (knife)} + R \text{ (hand-to-knife; contact)} + R \text{ (body)} + R \text{ (body-to-floor)} + R \text{ (earth return path)}.$$

Of all of these, the resistance of the earth, the knife and the mains supply are less than a few ohms and can be neglected. The body resistance (arm, trunk, leg) is about 300 Ω. The body contact resistances to the knife and to the floor will depend critically on the circumstances. Thus

$$I_{\text{shock}} = \frac{V}{R_{\text{body}} + R_{\text{contact}}}$$

A metal knife in a dry hand could lead to a contact resistance of about 5 kΩ. On a dry wooden floor, the body-to-floor resistance could exceed 100 kΩ, giving a total resistance of around 100 kΩ so

$$I_{\text{shock}} \approx \frac{240\text{ V}}{100\text{ k}\Omega}$$
$$\approx 2\text{ mA} \quad \text{i.e.} \quad \text{a few mA}$$

Our breakfaster would feel tingling, but would probably be unharmed.

However, in circumstances where the skin resistance is low (damp palms, wet feet on concrete etc.) the body contact resistance can easily fall below a few thousand ohms, and contact with a 240 volt active lead can give a fatal shock (> 100 mA).

## Earthed electrical equipment

Much electrical equipment (particularly in industry and in laboratories) is protected by connecting the outer metallic case to earth. In addition all active and neutral wiring is insulated. This scheme provides a two stage protection mechanism against electric shock. Figure 8.10 is a schematic of such a piece of apparatus.

If the active insulation breaks down, the fault current flows back to the source of power via the outer case and local ground (not through the neutral as usual).

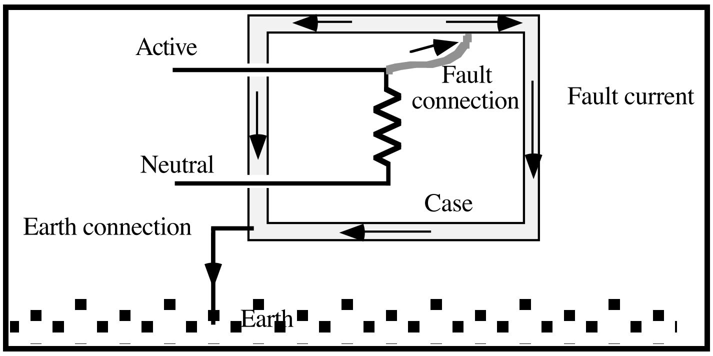


**Figure 8.10 An earthed case**

Earthed metal case of equipment is safe to touch even when the fault occurs.

Danger arises if the earth connection is not sound. The apparatus will continue to work, and it may not be noticed for years that the earth is broken. Now, however, if the insulation of the active wire fails, the case can become connected to the active lead. The equipment is in a condition which is **READY TO ELECTROCUTE** (figure 8.11).

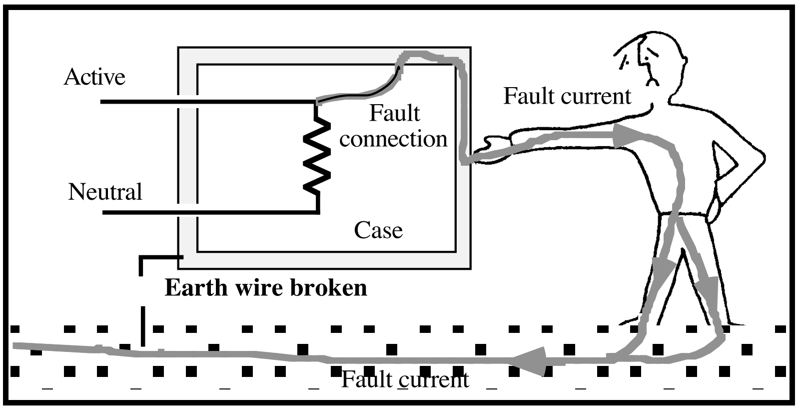


**Figure 8.11 Failure of an earth connection**

The metal casing, which was a safety feature when it was earthed, has become a means of delivering a fault current through the victim, along the path shown in figure 8.11.

The degree of danger depends on the skin contact resistance, as explained in the example above. One particularly dangerous situation arises when someone is connecting two pieces of equipment together, one of which is correctly earthed, and the other has a broken earth connection and an insulation fault. The fault current can then flow from one hand to the other (and a dry floor is of no help).

### Wiring convention

The usual Australian power point is referred to technically as a **g**eneral **p**urpose **o**utlet (or GPO). The diagram shows the wiring conventions for the active, neutral and earth conductors, as seen looking at the outlet.

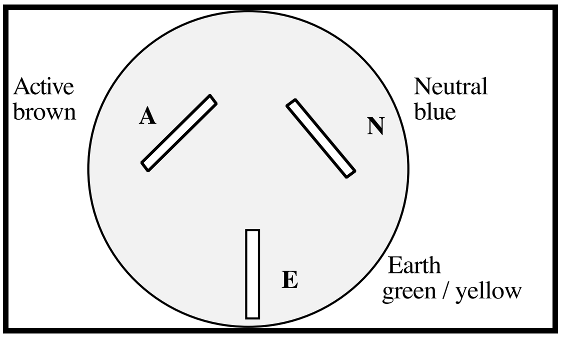


**Figure 8.12 Standard electrical socket**

The wiring and testing of plug connections should be done by licensed electricians, or other technically qualified personnel.

### Double insulation

Many small power tools and domestic appliances are double insulated. This means that instead of having an earthed conducting case, they have a plastic case and all external metal parts are separated from the mains by **two separate electrically insulating barriers** or a single insulator of equivalent insulating properties. In electric hand drills for example, part of the internal drive shaft is plastic.

Such equipment is marked with a double square symbol: ⧈.

### Microshock

In cardiac intensive care, patients may have catheters and electrodes inserted into or near the heart. Such patients are vulnerable to **microshock** produced by current accidentally carried directly to the heart by the inserted lines. Research at the Prince Henry Hospital in Sydney during the 1970s established that the minimum current passed directly into the heart which would induce ventricular fibrillation was only about 60 $\mu$A. Since a typical resistance of the saline column in a cardiac catheter is about 500 $\Omega$, it follows from that a potential as low as 30 mV on the catheter is potentially fatal.

Such small potentials are easily produced in any number of ways - contact potentials, earth leakage currents, induced EMF's, electrostatic effects, etc. Before the dangers were fully appreciated there is little doubt that many cardiac patients who died of heart failure were indeed victims of microshock.

The practices now adopted to avoid microshock in cardiac treatment areas are among the most rigorous protections against stray currents and potentials in the application of electricity. A very strict code of practice includes:

- high resistance electrodes and catheters where possible;

- a physically massive common earth conductor, known as the equipotential earth to which all equipment associated with the patient is connected (this even includes the bedside lamp);
- isolating transformers so that neither current carrying conductor is directly connected to the earth;
- automatic cut-outs which rapidly switch off the supply if a fault current is detected.

(For full details on this topic, consult Australian Standard No. AS2500.)

**Signs used in patient treatment areas**

Two grades of protection are used, each identified by standard signs as shown in figures 8.13 and 8.14.

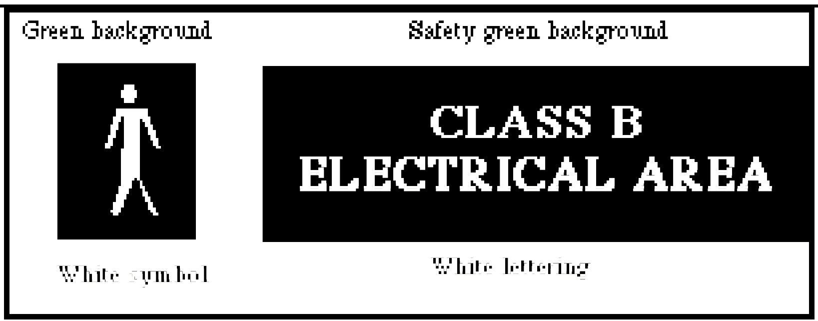


**Figure 8.13 Hospital signs**

Class B electrical area: patient connected to electro-medical equipment, but no direct connection made to the heart

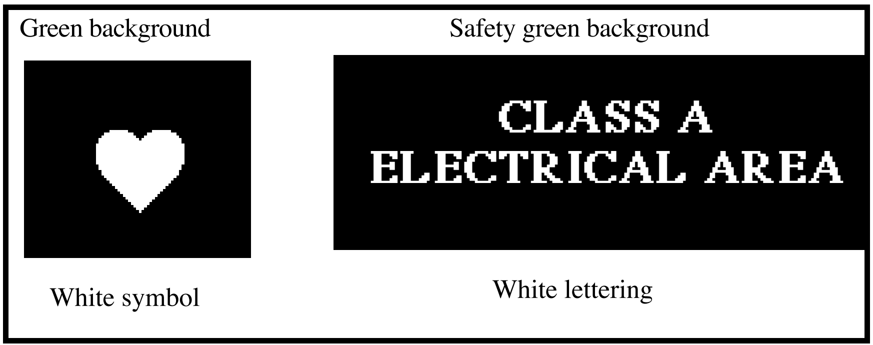


**Figure 8.14 Hospital signs**

Class A electrical area: electro-medical equipment or catheters making direct connection to the heart