

Question 1:

Fill in the blanks

- (i) The centre of a circle lies in _____ of the circle. (exterior/interior)
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in _____ of the circle. (exterior/interior)
- (iii) The longest chord of a circle is a _____ of the circle.
- (iv) An arc is a _____ when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and _____ of the circle.
- (vi) A circle divides the plane, on which it lies, in _____ parts.

Solution 1:

- (i) The centre of a circle lies in interior of the circle.
 - (ii) A point, whose distance from the centre of a circle is greater than its radius lies in exterior of the circle.
 - (iii) The longest chord of a circle is a diameter of the circle.
 - (iv) An arc is a semi-circle when its ends are the ends of a diameter.
 - (v) Segment of a circle is the region between an arc and chord of the circle.
 - (vi) A circle divides the plane, on which it lies, in three parts.
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Question 2:

Write True or False: Give reasons for your answers.

- (i) Line segment joining the centre to any point on the circle is a radius of the circle.
- (ii) A circle has only finite number of equal chords.
- (iii) If a circle is divided into three equal arcs, each is a major arc.
- (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
- (v) Sector is the region between the chord and its corresponding arc.
- (vi) A circle is a plane figure.

Solution 2:

- (i) **True.**

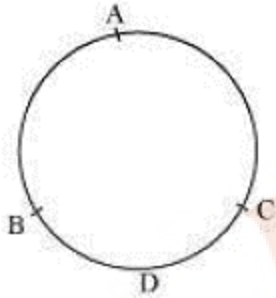
All the points on the circle are at equal distances from the centre of the circle, and this equal distance is called as radius of the circle.

- (ii) **False**

There are infinite points on a circle. Therefore, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.

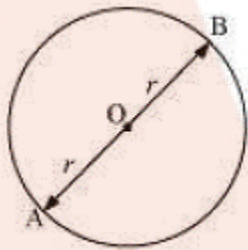
- (iii) **False**

Consider three arcs of same length as AB, BC, and CA. It can be observed that for minor arc BDC, CAB is a major arc. Therefore, AB, BC, and CA are minor arcs of the circle.



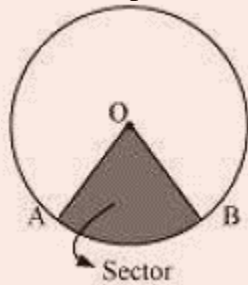
(iv) **True.**

Let AB be a chord which is twice as long as its radius. It can be observed that in this situation, our chord will be passing through the centre of the circle. Therefore, it will be the diameter of the circle.



(v) **False.**

Sector is the region between an arc and two radii joining the centre to the end points of the arc. For example, in the given figure, OAB is the sector of the circle.



(vi) **True.**

A circle is a two-dimensional figure and it can also be referred to as a plane figure.

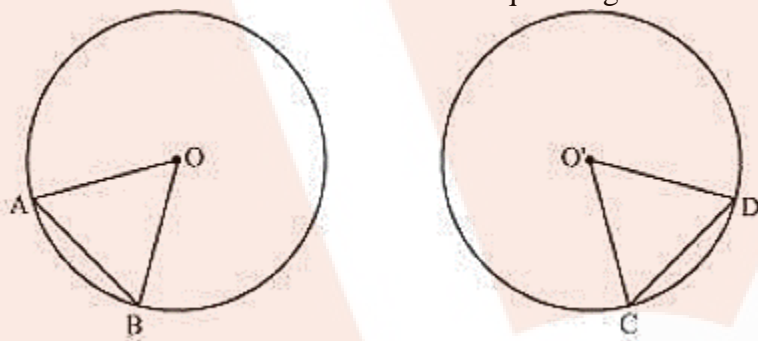
Exercise (10.2)

Question 1:

Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Solution 1:

A circle is a collection of points which are equidistant from a fixed point. This fixed point is called as the centre of the circle and this equal distance is called as radius of the circle. And thus, the shape of a circle depends on its radius. Therefore, it can be observed that if we try to superimpose two circles of equal radius, then both circles will cover each other. Therefore, two circles are congruent if they have equal radius. Consider two congruent circles having centre O and O' and two chords AB and CD of equal lengths.



In $\triangle AOB$ and $\triangle CO'D$,
 $AB = CD$ (Chords of same length)
 $OA = O'C$ (Radii of congruent circles)
 $OB = O'D$ (Radii of congruent circles)
 $\therefore \triangle AOB \cong \triangle CO'D$ (SSS congruence rule)
 $\Rightarrow \angle AOB = \angle CO'D$ (By CPCT)

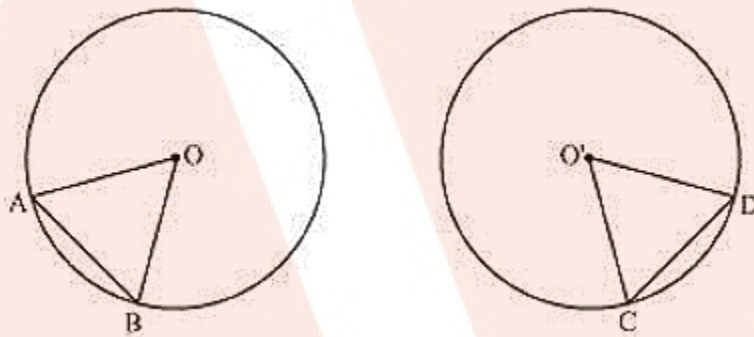
Hence, equal chords of congruent circles subtend equal angles at their centres.

Question 2:

Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Solution 2:

Let us consider two congruent circles (circles of same radius) with centres as O and O'.



In $\triangle AOB$ and $\triangle CO'D$,
 $\angle AOB = \angle CO'D$ (Given)
 $OA = O'C$ (Radii of congruent circles)
 $OB = O'D$ (Radii of congruent circles)
 $\therefore \triangle AOB \cong \triangle CO'D$ (SSS congruence rule)
 $\Rightarrow AB = CD$ (By CPCT)

Hence, if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

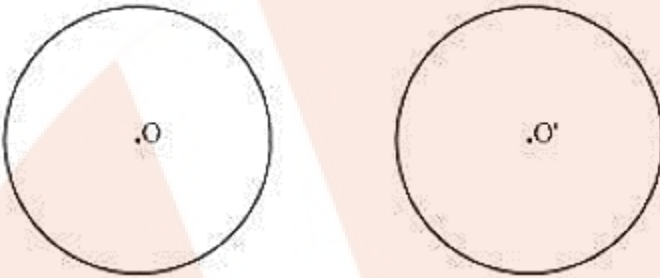
Exercise (10.3)

Question 1:

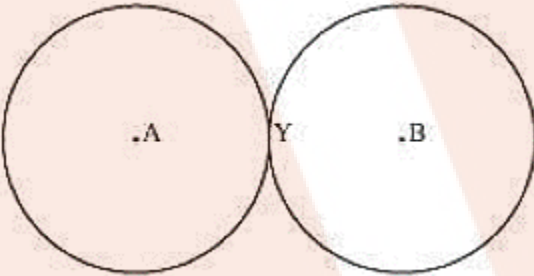
Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Solution 1:

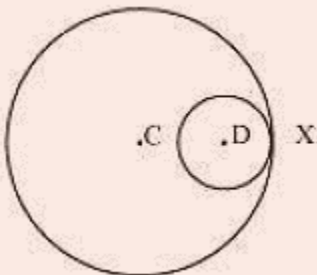
Consider the following pair of circles.



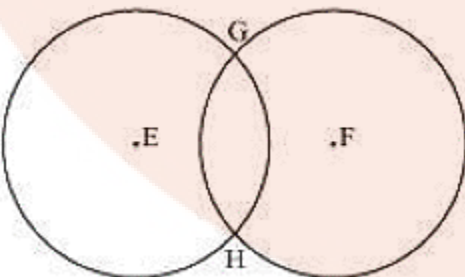
The above circles do not intersect each other at any point. Therefore, they do not have any point in common.



The above circles touch each other only at one point Y. Therefore, there is 1 point in common.



The above circles touch each other at 1 point X only. Therefore, the circles have 1 point in common.



These circles intersect each other at two points G and H. Therefore, the circles have two points in common. It can be observed that there can be a maximum of 2 points in common. Consider the situation in which two congruent circles are superimposed on each other. This situation can be referred to as if we are drawing the circle two times.

Question 2:

Suppose you are given a circle. Give a construction to find its centre.

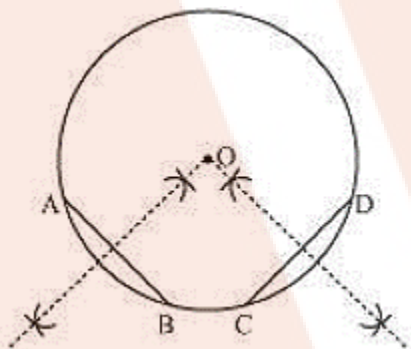
Solution 2:

The below given steps will be followed to find the centre of the given circle.

Step 1: Take the given circle.

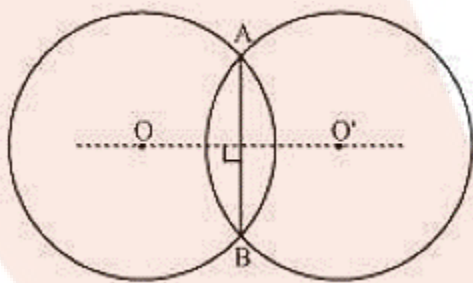
Step 2: Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.

Step 3: Let these perpendicular bisectors meet at point O. Hence, O is the centre of the given circle.



Question 3:

If two circles intersect at two points, then prove that their centres lie on the perpendicular bisector of the common chord.



Solution 3:

Consider two circles centered at point O and O', intersecting each other at point A and B respectively.

Join AB. AB is the chord of the circle centered at O. Therefore, perpendicular bisector of AB will pass through O.

Again, AB is also the chord of the circle centered at O'. Therefore, perpendicular bisector of AB will also pass through O'.

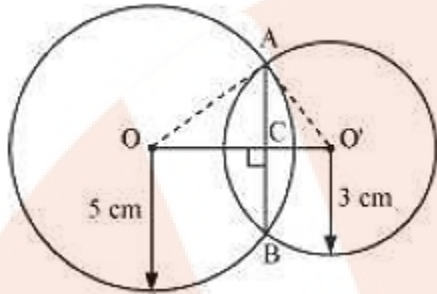
Clearly, the centres of these circles lie on the perpendicular bisector of the common chord.

Exercise (10.4)

Question 1:

Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution 1:



Let the radius of the circle centered at O and O' be 5 cm and 3 cm respectively.

$$OA = OB = 5 \text{ cm}$$

$$O'A = O'B = 3 \text{ cm}$$

OO' will be the perpendicular bisector of chord AB.

$$\therefore AC = CB$$

It is given that, $OO' = 4 \text{ cm}$

Let OC be x. Therefore, O'C will be $4 - x$.

In $\triangle OAC$,

$$OA^2 = AC^2 + OC^2$$

$$\Rightarrow 5^2 = AC^2 + x^2$$

$$\Rightarrow 25 - x^2 = AC^2 \quad \dots (1)$$

In $\triangle O'AC$,

$$O'A^2 = AC^2 + O'C^2$$

$$\Rightarrow 3^2 = AC^2 + (4 - x)^2$$

$$\Rightarrow 9 = AC^2 + 16 + x^2 - 8x$$

$$\Rightarrow AC^2 = -x^2 - 7 + 8x \quad \dots (2)$$

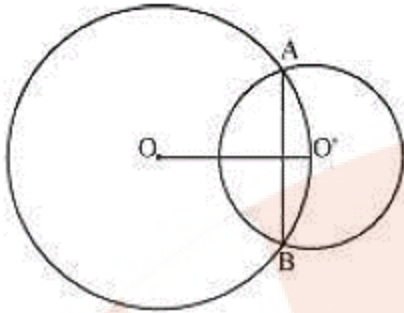
From Equations (1) and (2), we obtain

$$25 - x^2 = -x^2 - 7 + 8x$$

$$8x = 32$$

$$x = 4$$

Therefore, the common chord will pass through the centre of the smaller circle i.e., O' and hence, it will be the diameter of the smaller circle.



$$AC^2 = 25 - x^2 = 25 - 16 = 9$$

$$\therefore AC = 3 \text{ cm}$$

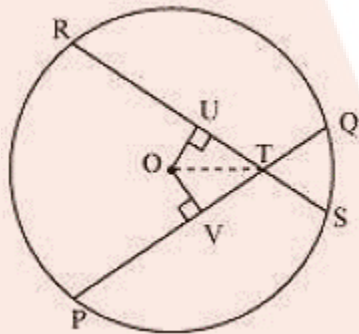
$$\text{Length of the common chord } AB = 2AC = (2 \times 3) \text{ m} = 6 \text{ cm}$$

Question 2:

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution 2:

Let PQ and RS be two equal chords of a given circle and they are intersecting each other at point T.



Draw perpendiculars OV and OU on these chords.

In $\triangle OVT$ and $\triangle OUT$,

$OV = OU$ (Equal chords of a circle are equidistant from the centre)

$\angle OVT = \angle OUT$ (Each 90°)

$OT = OT$ (Common)

$\therefore \triangle OVT \cong \triangle OUT$ (RHS congruence rule)

$\therefore VT = UT$ (By CPCT) ... (1)

It is given that,

$$PQ = RS \quad \dots (2)$$

$$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$$

$$\Rightarrow PV = RU \quad \dots (3)$$

On adding Equations (1) and (3), we obtain

$$PV + VT = RU + UT$$

$$\Rightarrow PT = RT \quad \dots (4)$$

On subtracting Equation (4) from Equation (2), we obtain

$$PQ - PT = RS - RT$$

$$\Rightarrow QT = ST \quad \dots (5)$$

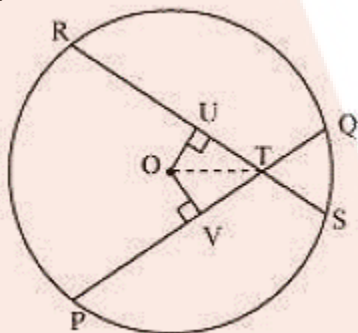
Equations (4) and (5) indicate that the corresponding segments of chords PQ and RS are congruent to each other.

Question 3:

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution 3:

Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T.



Draw perpendiculars OV and OU on these chords.

In $\triangle OVT$ and $\triangle OUT$,

$OV = OU$ (Equal chords of a circle are equidistant from the centre)

$\angle OVT = \angle OUT$ (Each 90°)

$OT = OT$ (Common)

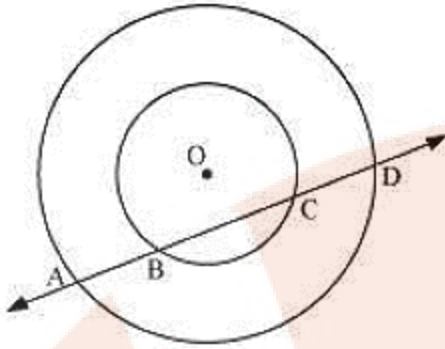
$\therefore \triangle OVT \cong \triangle OUT$ (RHS congruence rule)

$\therefore \angle OTV = \angle OTU$ (By CPCT)

Therefore, it is proved that the line joining the point of intersection to the centre makes equal angles with the chords.

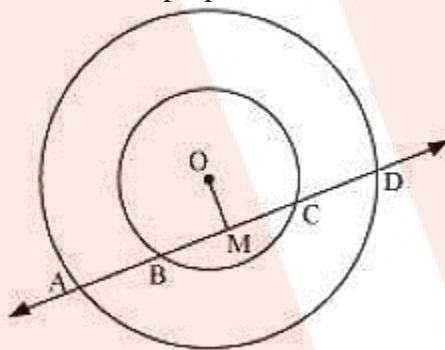
Question 4:

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see figure).



Solution 4:

Let us draw a perpendicular OM on line AD.



It can be observed that BC is the chord of the smaller circle and AD is the chord of the bigger circle.

We know that perpendicular drawn from the centre of the circle bisects the chord.

$$\therefore BM = MC \quad \dots (1)$$

$$\text{And, } AM = MD \quad \dots (2)$$

On subtracting Equation (2) from (1), we obtain

$$AM - BM = MD - MC$$

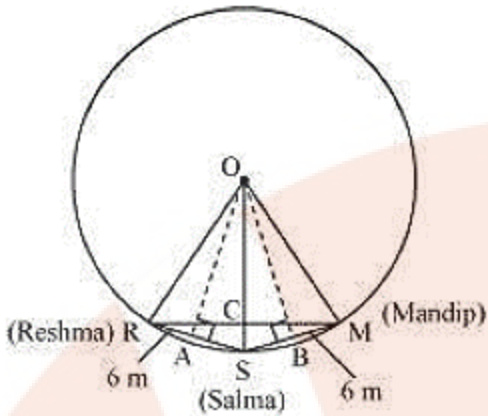
$$\therefore AB = CD$$

Question 5:

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Solution 5:

Draw perpendiculars OA and OB on RS and SM respectively.



$$AR = AS = \frac{6}{2} = 3 \text{ m}$$

$OR = OS = OM = 5 \text{ m}$. (Radii of the circle)

In $\triangle OAR$,

$$OA^2 + AR^2 = OR^2$$

$$OA^2 + (3 \text{ m})^2 = (5 \text{ m})^2$$

$$OA^2 = (25 - 9) \text{ m}^2 = 16 \text{ m}^2$$

$$OA = 4 \text{ m}$$

ORSM will be a kite ($OR = OM$ and $RS = SM$). We know that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal.

$\angle RCS$ will be of 90° and $RC = CM$

$$\text{Area of } \triangle ORS = \frac{1}{2} \times OA \times RS$$

$$\frac{1}{2} \times RC \times OS = \frac{1}{2} \times 4 \times 6$$

$$RC \times 5 = 24$$

$$RC = 4.8$$

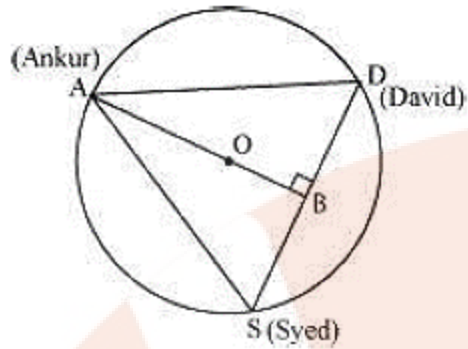
$$RM = 2RC = 2(4.8) = 9.6$$

Therefore, the distance between Reshma and Mandip is 9.6 m.

Question 6:

A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution 6:



It is given that $AS = SD = DA$

Therefore, $\triangle ASD$ is an equilateral triangle.

OA (radius) = 20 m

Medians of equilateral triangle pass through the circumcentre (O) of the equilateral triangle ASD. We also know that medians intersect each other in the ratio 2:1. As AB is the median of equilateral triangle ASD, we can write

$$\Rightarrow \frac{OA}{OB} = \frac{2}{1}$$

$$\Rightarrow \frac{20 \text{ m}}{OB} = \frac{2}{1}$$

$$\Rightarrow OB = \left(\frac{20}{2}\right) = 10 \text{ m}$$

$$AB = OA + OB = (20 + 10) \text{ m} = 30 \text{ m}$$

In $\triangle ABD$,

$$AD^2 = AB^2 + BD^2$$

$$AD^2 = (30)^2 + \left(\frac{AD}{2}\right)^2$$

$$AD^2 = 900 + \frac{1}{4}AD^2$$

$$\frac{3}{4}AD^2 = 900$$

$$AD^2 = 1200$$

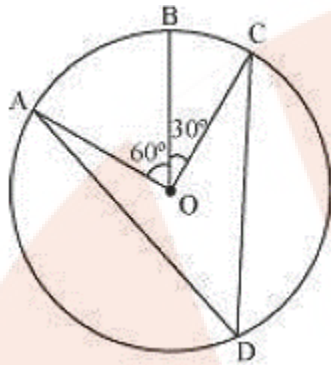
$$AD = 20\sqrt{3}$$

Therefore, the length of the string of each phone will be $20\sqrt{3}$ m.

Exercise (10.5)

Question 1:

In the given figure, A, B and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.



Solution 1:

It can be observed that

$$\begin{aligned}\angle AOC &= \angle AOB + \angle BOC \\ &= 60^\circ + 30^\circ \\ &= 90^\circ\end{aligned}$$

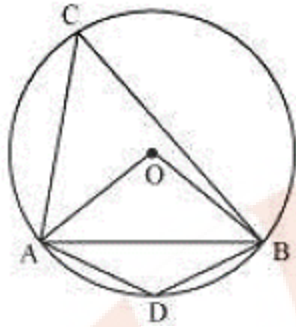
We know that angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$$\angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} (90^\circ) = 45^\circ$$

Question 2:

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution 2:



In $\triangle OAB$,

$AB = OA = OB = \text{radius}$

$\therefore \triangle OAB$ is an equilateral triangle.

Therefore, each interior angle of this triangle will be of 60° .

$\therefore \angle AOB = 60^\circ$

$$\angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} (60^\circ) = 30^\circ$$

In cyclic quadrilateral $ACBD$,

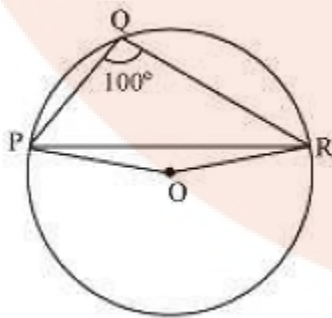
$\angle ACB + \angle ADB = 180^\circ$ (Opposite angle in cyclic quadrilateral)

$\therefore \angle ADB = 180^\circ - 30^\circ = 150^\circ$

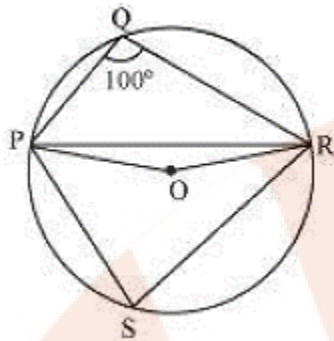
Therefore, angle subtended by this chord at a point on the major arc and the minor arc are 30° and 150° respectively.

Question 3:

In the given figure, $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Solution 3:



Consider PR as a chord of the circle.

Take any point S on the major arc of the circle.

PQRS is a cyclic quadrilateral

$\angle PQR + \angle PSR = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$$\therefore \angle PSR = 180^\circ - 100^\circ = 80^\circ$$

We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\therefore \angle POR = 2\angle PSR = 2(80^\circ) = 160^\circ$$

In $\triangle POR$,

$OP = OR$ (Radii of the same circle)

$\therefore \angle OPR = \angle ORP$ (Angles opposite to equal sides of a triangle)

$\angle OPR + \angle ORP + \angle POR = 180^\circ$ (Angle sum property of a triangle)

$$2\angle OPR + 160^\circ = 180^\circ$$

$$2\angle OPR = 180^\circ - 160^\circ = 20^\circ$$

$$\angle OPR = 10^\circ$$

Question 4:

In fig. 10.38, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$?

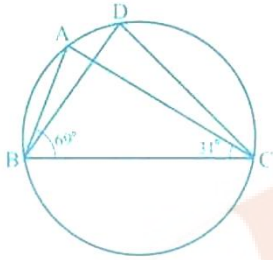


Fig. 10.38

Solution 4:

$\angle BAC = \angle BDC$ (angles in the same segment of the circle)

In $\triangle ABC$,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$ (Angle sum property of a \triangle)

Or, $\angle BAC + 69^\circ + 31^\circ = 180^\circ$

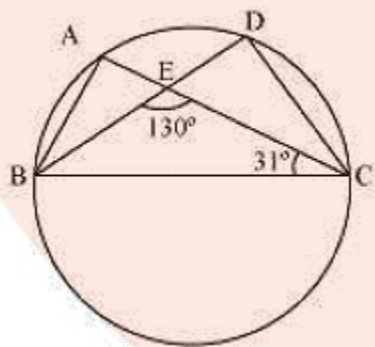
Or, $\angle BAC = 180^\circ - 100^\circ$

Or, $\angle BAC = 80^\circ$

Thus, $\angle BDC = 80^\circ$

Question 5:

In the given figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.



Solution 5:

In $\triangle CDE$,

$\angle CDE + \angle DCE = \angle CEB$ (Exterior angle)

$\therefore \angle CDE + 20^\circ = 130^\circ$

$$\therefore \angle CDE = 110^\circ$$

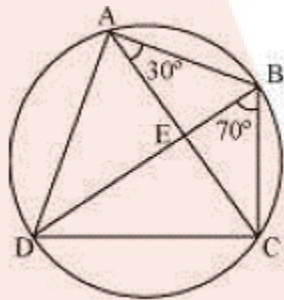
However, $\angle BAC = \angle CDE$ (Angles in the same segment of a circle)

$$\therefore \angle BAC = 110^\circ$$

Question 6:

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC$ is 30° , find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Solution 6:



For chord CD,

$\angle CBD = \angle CAD$ (Angles in the same segment)

$$\angle CAD = 70^\circ$$

$$\angle BAD = \angle BAC + \angle CAD = 30^\circ + 70^\circ = 100^\circ$$

$\angle BCD + \angle BAD = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$$\angle BCD + 100^\circ = 180^\circ$$

$$\angle BCD = 80^\circ$$

In $\triangle ABC$,

$AB = BC$ (Given)

$\therefore \angle BCA = \angle CAB$ (Angles opposite to equal sides of a triangle)

$$\therefore \angle BCA = 30^\circ$$

We have, $\angle BCD = 80^\circ$

$$\therefore \angle BCA + \angle ACD = 80^\circ$$

$$30^\circ + \angle ACD = 80^\circ$$

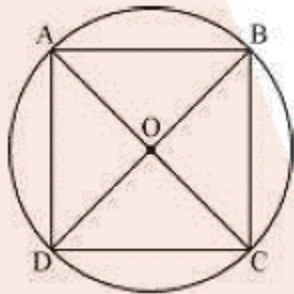
$$\therefore \angle ACD = 50^\circ$$

$$\therefore \angle ECD = 50^\circ$$

Question 7:

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution 7:



Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.

$$\angle BAD = \frac{1}{2} \angle BOD = \frac{180^\circ}{2} = 90^\circ \quad (\text{Consider BD as a chord})$$

$$\angle BCD + \angle BAD = 180^\circ \quad (\text{Cyclic quadrilateral})$$

$$\angle BCD = 180^\circ - 90^\circ = 90^\circ$$

$$\angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} (180^\circ) = 90^\circ \quad (\text{Considering AC as a chord})$$

$$\angle ADC + \angle ABC = 180^\circ \quad (\text{Cyclic quadrilateral})$$

$$90^\circ + \angle ABC = 180^\circ$$

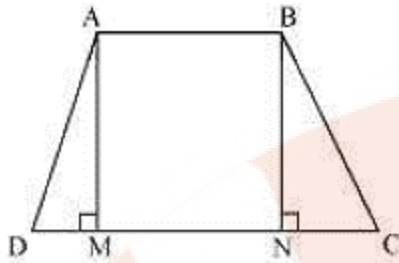
$$\angle ABC = 90^\circ$$

Each interior angle of a cyclic quadrilateral is of 90° . Hence, it is a rectangle.

Question 8:

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution 8:



Consider a trapezium ABCD with $AB \parallel CD$ and $BC = AD$.

Draw $AM \perp CD$ and $BN \perp CD$.

In $\triangle AMD$ and $\triangle BNC$,

$AD = BC$ (Given)

$\angle AMD = \angle BNC$ (By construction, each is 90°)

$AM = BN$ (Perpendicular distance between two parallel lines is same)

$\therefore \triangle AMD \cong \triangle BNC$ (RHS congruence rule)

$\therefore \angle ADC = \angle BCD$ (CPCT) ... (1)

$\angle BAD$ and $\angle ADC$ are on the same side of transversal AD.

$\angle BAD + \angle ADC = 180^\circ$... (2)

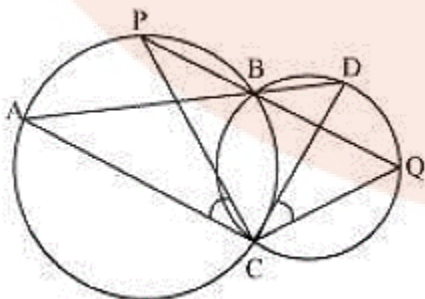
$\angle BAD + \angle BCD = 180^\circ$ [Using Equation (1)]

This equation shows that the opposite angles are supplementary.

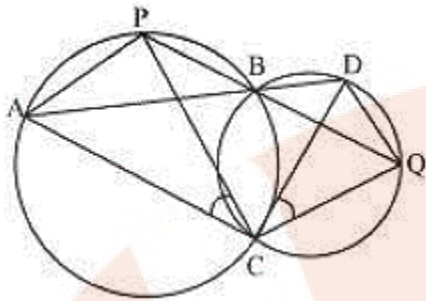
Therefore, ABCD is a cyclic quadrilateral.

Question 9:

Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see the given figure). Prove that $\angle ACP = \angle QCD$.



Solution 9:



Join chords AP and DQ.

For chord AP,

$$\angle PBA = \angle ACP \text{ (Angles in the same segment)} \quad \dots (1)$$

For chord DQ,

$$\angle DBQ = \angle QCD \text{ (Angles in the same segment)} \quad \dots (2)$$

ABD and PBQ are line segments intersecting at B.

$$\therefore \angle PBA = \angle DBQ \text{ (Vertically opposite angles)} \quad \dots (3)$$

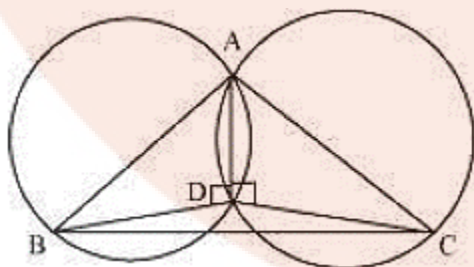
From Equations (1), (2), and (3), we obtain

$$\angle ACP = \angle QCD$$

Question 10:

If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution 10:



Consider a $\triangle ABC$.

Two circles are drawn while taking AB and AC as the diameter.

Let them intersect each other at D and let D not lie on BC.

Join AD.

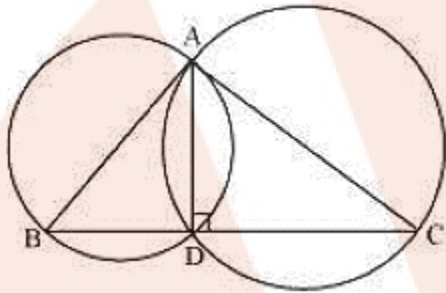
$$\angle ADB = 90^\circ \text{ (Angle subtended by semi-circle)}$$

$$\angle ADC = 90^\circ \text{ (Angle subtended by semi-circle)}$$

$$\angle BDC = \angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$$

Therefore, BDC is a straight line and hence, our assumption was wrong.

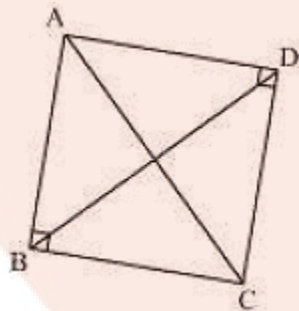
Thus, Point D lies on third side BC of $\triangle ABC$.



Question 11:

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Solution 11:



In $\triangle ABC$,

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$90^\circ + \angle BCA + \angle CAB = 180^\circ$$

$$\angle BCA + \angle CAB = 90^\circ \quad \dots (1)$$

In $\triangle ADC$,

$$\angle CDA + \angle ACD + \angle DAC = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$90^\circ + \angle ACD + \angle DAC = 180^\circ$$

$$\angle ACD + \angle DAC = 90^\circ \quad \dots (2)$$

Adding Equations (1) and (2), we obtain

$$\angle BCA + \angle CAB + \angle ACD + \angle DAC = 180^\circ$$

$$(\angle BCA + \angle ACD) + (\angle CAB + \angle DAC) = 180^\circ$$

$$\angle BCD + \angle DAB = 180^\circ \quad \dots (3)$$

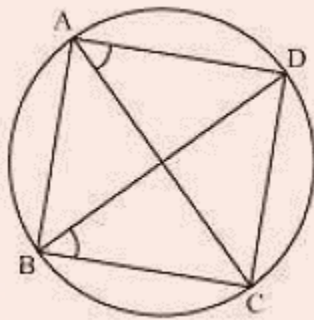
However, it is given that

$$\angle B + \angle D = 90^\circ + 90^\circ = 180^\circ \quad \dots (4)$$

From Equations (3) and (4), it can be observed that the sum of the measures of opposite angles of quadrilateral ABCD is 180° . Therefore, it is a cyclic quadrilateral.

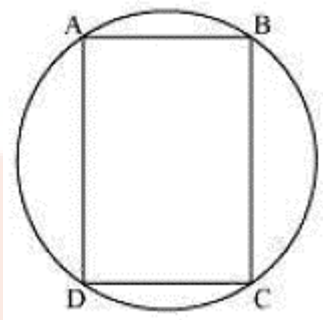
Consider chord CD.

$$\angle CAD = \angle CBD \text{ (Angles in the same segment)}$$



Question 12:

Prove that a cyclic parallelogram is a rectangle.

Solution 12:

Let ABCD be a cyclic parallelogram.

$$\angle A + \angle C = 180^\circ \text{ (Opposite angles of a cyclic quadrilateral)} \quad \dots (1)$$

We know that opposite angles of a parallelogram are equal.

$$\angle A = \angle C \text{ and } \angle B = \angle D$$

From Equation (1),

$$\angle A + \angle C = 180^\circ$$

$$\angle A + \angle A = 180^\circ$$

$$2\angle A = 180^\circ$$

$$\angle A = 90^\circ$$

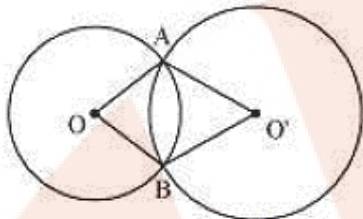
Parallelogram ABCD has one of its interior angles as 90° . Therefore, it is a rectangle.

Exercise (10.6)

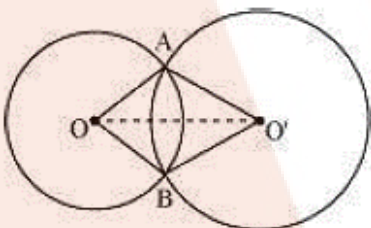
Question 1:

Prove that line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution 1:



Let two circles having their centres as O and O' intersect each other at point A and B respectively. Let us join O'.



In $\triangle AOO'$ and $\triangle BOO'$,

$OA = OB$ (Radius of circle 1)

$O'A = O'B$ (Radius of circle 2)

$OO' = OO'$ (Common)

$\therefore \triangle AOO' \cong \triangle BOO'$ (By SSS congruence rule)

$\angle OAO' = \angle OBO'$ (By CPCT)

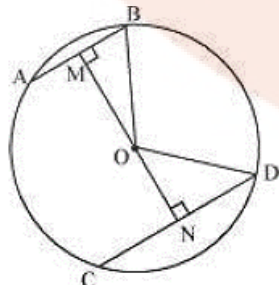
Therefore, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Question 2:

Two chords AB and CD of lengths 5 cm 11cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution 2:

Draw $OM \perp AB$ and $ON \perp CD$. Join OB and OD.



$$BM = \frac{AB}{2} = \frac{5}{2} \quad (\text{Perpendicular from the centre bisects the chord})$$

$$ND = \frac{CD}{2} = \frac{11}{2}$$

Let ON be x. Therefore, OM will be 6 - x.

In ΔMOB ,

$$OM^2 + MB^2 = OB^2$$

$$(6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$$

$$36 + x^2 - 12x + \frac{25}{4} = OB^2 \quad \dots (1)$$

In ΔNOD ,

$$ON^2 + ND^2 = OD^2$$

$$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$$

$$x^2 + \frac{121}{4} = OD^2 \quad \dots (2)$$

We have $OB = OD$ (Radii of the same circle)

Therefore, from Equations (1) and (2),

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$$

$$x = 1$$

From Equation (2),

$$(1)^2 + \frac{121}{4} = OD^2$$

$$OD^2 = \frac{121}{4} + 1 = \frac{125}{4}$$

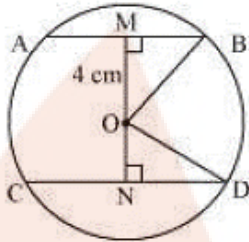
$$OD = \frac{5}{2}\sqrt{5}$$

Therefore, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm. = 5.6 cm (approx.)

Question 3:

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution 3:



Let AB and CD be two parallel chords in a circle centered at O. Join OB and OD.

Distance of smaller chord AB from the centre of the circle = 4 cm

$$OM = 4 \text{ cm}$$

$$MB = \frac{AB}{2} = \frac{6}{2} = 3 \text{ cm}$$

In $\triangle OMB$,

$$OM^2 + MB^2 = OB^2$$

$$(4)^2 + (3)^2 = OB^2$$

$$16 + 9 = OB^2$$

$$OB = \sqrt{25}$$

$$OB = 5 \text{ cm}$$

In $\triangle OND$,

$$OD = OB = 5 \text{ cm} \quad (\text{Radii of the same circle})$$

$$ND = \frac{CD}{2} = \frac{8}{2} = 4 \text{ cm}$$

$$ON^2 + ND^2 = OD^2$$

$$ON^2 + (4)^2 = (5)^2$$

$$ON^2 = 25 - 16 = 9$$

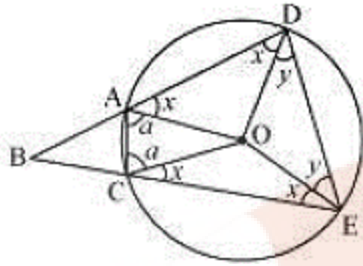
$$ON = 3 \text{ cm}$$

Therefore, the distance of the bigger chord from the centre is 3 cm.

Question 4:

Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Solution 4:



In $\triangle AOD$ and $\triangle COE$,
 $OA = OC$ (Radii of the same circle)
 $OD = OE$ (Radii of the same circle)
 $AD = CE$ (Given)

$\therefore \triangle AOD \cong \triangle COE$ (SSS congruence rule)

$$\angle OAD = \angle OCE \text{ (By CPCT)} \quad \dots (1)$$

$$\angle ODA = \angle OEC \text{ (By CPCT)} \quad \dots (2)$$

Also,

$$\angle OAD = \angle ODA \text{ (As } OA = OD) \quad \dots (3)$$

From Equations (1), (2), and (3), we obtain

$$\angle OAD = \angle OCE = \angle ODA = \angle OEC$$

$$\text{Let } \angle OAD = \angle OCE = \angle ODA = \angle OEC = x$$

In $\triangle OAC$,

$$OA = OC$$

$$\therefore \angle OCA = \angle OAC \text{ (Let } a)$$

In $\triangle ODE$,

$$OD = OE$$

$$\angle OED = \angle ODE \text{ (Let } y)$$

ADEC is a cyclic quadrilateral.

$$\therefore \angle CAD + \angle DEC = 180^\circ \text{ (Opposite angles are supplementary)}$$

$$x + a + x + y = 180^\circ$$

$$2x + a + y = 180^\circ$$

$$y = 180^\circ - 2x - a \quad \dots (4)$$

$$\text{However, } \angle DOE = 180^\circ - 2y$$

$$\text{And, } \angle AOC = 180^\circ - 2a$$

$$\angle DOE - \angle AOC = 2a - 2y = 2a - 2(180^\circ - 2x - a)$$

$$= 4a + 4x - 360^\circ \quad \dots (5)$$

$$\angle BAC + \angle CAD = 180^\circ \text{ (Linear pair)}$$

$$\therefore \angle BAC = 180^\circ - \angle CAD = 180^\circ - (a + x)$$

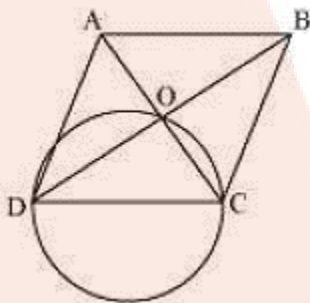
$$\text{Similarly, } \angle ACB = 180^\circ - (a + x)$$

In $\triangle ABC$,
 $\angle ABC + \angle BAC + \angle ACB = 180^\circ$ (Angle sum property of a triangle)
 $\angle ABC = 180^\circ - \angle BAC - \angle ACB$
 $= 180^\circ - (180^\circ - a - x) - (180^\circ - a - x)$
 $= 2a + 2x - 180^\circ$
 $= \frac{1}{2}[4a + 4x - 360^\circ]$
 $\angle ABC = \frac{1}{2}[\angle DOE - \angle AOC]$ [Using Equation (5)]

Question 5:

Prove that the circle drawn with any side of a rhombus as diameter passes through the point of intersection of its diagonals.

Solution 5:



Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn while taking side CD as its diameter. We know that a diameter subtends 90° on the arc.

$$\therefore \angle COD = 90^\circ$$

Also, in rhombus, the diagonals intersect each other at 90° .

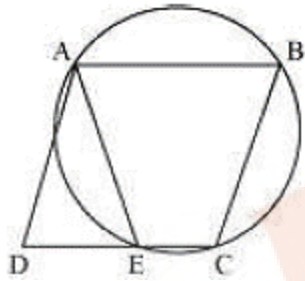
$$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$$

Clearly, point O has to lie on the circle.

Question 6:

ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that $AE = AD$.

Solution 6:



It can be observed that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral, the sum of the opposite angles is 180° .

$$\angle AEC + \angle CBA = 180^\circ$$

$$\angle AEC + \angle AED = 180^\circ \text{ (Linear pair)}$$

$$\angle AED = \angle CBA \quad \dots (1)$$

For a parallelogram, opposite angles are equal.

$$\angle ADE = \angle CBA \quad \dots (2)$$

From (1) and (2),

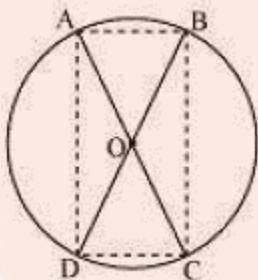
$$\angle AED = \angle ADE$$

$AD = AE$ (sides opposite to equal Angles of a triangle are equal).

Question 7:

AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

Solution 7:



Let two chords AB and CD are intersecting each other at point O.

In $\triangle AOB$ and $\triangle COD$,

$$OA = OC \text{ (Given)}$$

$$OB = OD \text{ (Given)}$$

$$\angle AOB = \angle COD \text{ (Vertically opposite angles)}$$

$$\therefore \triangle AOB \cong \triangle COD \text{ (SAS congruence rule)}$$

$$AB = CD \text{ (By CPCT)}$$

Similarly, it can be proved that $\triangle AOD \cong \triangle COB$

$\therefore AD = CB$ (By CPCT)

Since in quadrilateral ACBD, opposite sides are equal in length, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

$\therefore \angle A = \angle C$

However, $\angle A + \angle C = 180^\circ$ (ABCD is a cyclic quadrilateral)

$\angle A + \angle A = 180^\circ$

$2\angle A = 180^\circ$

$\therefore \angle A = 90^\circ$

As ACBD is a parallelogram and one of its interior angles is 90° , therefore, it is a rectangle.

$\angle A$ is the angle subtended by chord BD.

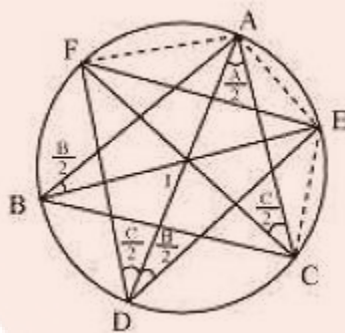
And as $\angle A = 90^\circ$, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

Question 8:

Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F

respectively. Prove that the angles of the triangle DEF are $90^\circ - \frac{1}{2}A$, $90^\circ - \frac{1}{2}B$, and $90^\circ - \frac{1}{2}C$.

Solution 8:



It is given that BE is the bisector of $\angle B$.

$\therefore \angle ABE = \frac{\angle B}{2}$

However, $\angle ADE = \angle ABE$ (Angles in the same segment for chord AE)

$\therefore \angle ADE = \frac{\angle B}{2}$

Similarly, $\angle ACF = \angle ADF = \frac{\angle C}{2}$ (Angle in the same segment for chord AF)

$\angle D = \angle ADE + \angle ADF$

$$\begin{aligned}
 &= \frac{\angle B}{2} + \frac{\angle C}{2} \\
 &= \frac{1}{2}(\angle B + \angle C) \\
 &= \frac{1}{2}(180^\circ - \angle A) \\
 &= 90^\circ - \frac{1}{2}\angle A
 \end{aligned}$$

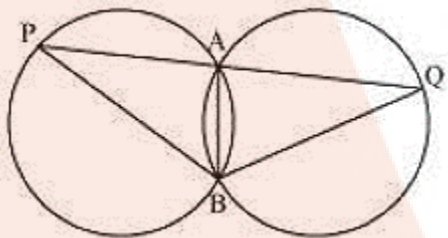
Similarly, it can be proved that

$$\begin{aligned}
 \angle E &= 90^\circ - \frac{1}{2}\angle B \\
 \angle F &= 90^\circ - \frac{1}{2}\angle C
 \end{aligned}$$

Question 9:

Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Solution 9:



AB is the common chord in both the congruent circles.

$$\therefore \angle APB = \angle AQB$$

In $\triangle BPQ$,

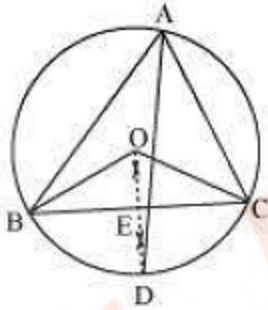
$$\angle APB = \angle AQB$$

$$\therefore BQ = BP \text{ (Sides opposite to equal angles of a triangle are equal)}$$

Question 10:

In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circum-circle of the triangle ABC.

Solution 10:



Let perpendicular bisector of side BC and angle bisector of $\angle A$ meet at point D. Let the perpendicular bisector of side BC intersect it at E.

Perpendicular bisector of side BC will pass through circumcentre O of the circle. $\angle BOC$ and

$\angle BAC$ are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively. We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle BOC = 2\angle BAC = 2\angle A \quad \dots (1)$$

In $\triangle BOE$ and $\triangle COE$,

$OE = OE$ (Common)

$OB = OC$ (Radii of same circle)

$\angle OEB = \angle OEC$ (Each 90° as $OD \perp BC$)

$\therefore \triangle BOE \cong \triangle COE$ (RHS congruence rule)

$\angle BOE = \angle COE$ (By CPCT) $\dots (2)$

However, $\angle BOE + \angle COE = \angle BOC$

$\therefore \angle BOE + \angle BOE = 2\angle A$ [Using Equations (1) and (2)]

$$2\angle BOE = 2\angle A$$

$$\angle BOE = \angle A$$

$$\angle BOE = \angle COE = \angle A$$

The perpendicular bisector of side BC and angle bisector of $\angle A$ meet at point D.

$$\therefore \angle BOD = \angle BOE = \angle A \quad \dots (3)$$

Since AD is the bisector of angle $\angle A$,

$$\angle BAD = \frac{\angle A}{2}$$

$$\therefore 2\angle BAD = \angle A \quad \dots (4)$$

From Equations (3) and (4), we obtain

$$\angle BOD = 2\angle BAD$$

This can be possible only when point D will be a chord of the circle. For this, the point D lies on the circumcircle.

Therefore, the perpendicular bisector of side BC and the angle bisector of $\angle A$ meet on the circumcircle of triangle ABC.

